MEASURE ALGEBRAS ON INFINITE DIMENSIONAL SPACES

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Hewitt and Zuckerman [3] studied the measure algebra of a compact interval and Ross [7] extended these results to locally compact intervals. Baartz [1] then considered finite Cartesian products of intervals; however, he demonstrated the impossibility of extending his results to infinite Cartesian products. The purpose of this article is to extend the above works to infinite dimensional spaces by using weak products, rather than Cartesian products. The maximal ideal space of the measure algebra will be identified, the Gelfand topology described, and many Banach algebra type results, such as semisimplicity, regularity, Choquet boundary, etc., will be investigated. Finally, a Herglotz-Bochner theorem will be obtained.

1. Preliminaries

Let S be a totally ordered set, with a least element 0. We make S into a topological semigroup by putting on the interval topology and defining multiplication by $xy = \max(x, y)$. We shall assume that S is compact, and write $S = [0, 1]$; however, the extension of the results of this article to the locally compact case can be accomplished in a manner precisely the same as in Ross [7]. If $\{S_\gamma \mid \gamma \in \Gamma\}$ is a collection of these *order intervals*, we define the weak product,

$$\begin{aligned} S &= \prod^{w} \{S_\gamma \mid \gamma \in \Gamma\} \\ &= \{(x_\gamma) \in \prod \{S_\gamma \mid \gamma \in \Gamma\} \mid x_\gamma = 0_\gamma \text{ for all but finitely many coordinates}\}. \end{aligned}$$

A basis for the topology on S is given by

$$\left\{\prod^{w} \{U_\gamma \mid \gamma \in \Gamma\} \mid U_\gamma \text{ open in } S_\gamma,\, 0_\gamma \in U_\gamma \text{ for all but finitely many coordinates}\right\}.$$

Multiplication and order is coordinatewise. Finally

$$L_x = \{y \in S \mid y \leq x\} \quad \text{and} \quad M_x = \{y \in S \mid y \geq x\}.$$

We let $M(S)$ denote the space of all bounded, regular Borel measures on S, equipped with the total variation norm. We make $M(S)$ into a Banach algebra by defining

$$\mu * \nu(E) = \int_S \int_S \chi_E(xy)\, d\mu(x)\, d\nu(y), \quad \mu, \nu \in M(S),$$

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