A MONOTONE INTERSECTION PROPERTY FOR MANIFOLDS

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1. Introduction

Brown [1] has established that if N is an open n-cell, then N has the monotone union property. Kwun [2] proved that if N is a closed PL manifold whose dimension is not four, then $M - p$ has the monotone union property where p is any point of M. In this paper we establish conditions which tell us when a manifold has the monotone union property. We define a monotone intersection property and indicate ways that it is related to the monotone union property. The principal results established are:

THEOREM 2.3. *Let $\{N_i\}$ be a sequence of manifolds such that for each i ($i = 1, 2, \ldots$), N_i is trivially embedded in N_{i+1}. Then $\bigcup_{i=1}^{\infty} N_i$ is homeomorphic to N_1°.*

THEOREM 3.5. *A manifold N has the monotone intersection property if and only if whenever $N \subset^\circ N_1$ where N_1 is homeomorphic to N, then N is trivially embedded in N_1.*

COROLLARY 3.6. *If a manifold has the monotone intersection property, then it also has the monotone union property.*

Theorem 2.3 generalizes the following result of C. H. Edwards [3] to all compact manifolds with boundary. Let N be a compact 3-manifold with boundary B and spine K and for each integer n let $h_n(N)$ be a homeomorphic image of N. Edwards has shown that if $X = \bigcup_{i=1}^{\infty} h_n(N)$ where for each n, $h_n(N) \subset^\circ h_{n+1}(N)$ and $h_n(K) = h_{n+1}(K)$, then X is homeomorphic to N°. Husch [4] stated that this result could be extended to all PL manifolds by using the regular neighborhood annulus conjecture.

The following question is raised by this paper. Are the monotone union and monotone intersection properties equivalent for compact topological manifolds with boundary?

2. Definitions and proof of Theorem 2.3

Throughout this paper we assume all manifolds are compact topological manifolds with boundary and all homeomorphisms are topological. A compact

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