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THE TOPOLOGICAL STRUCTURE OF 4-MANIFOLDS WITH EFFECTIVE TORUS ACTIONS (II)

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A closed orientable 4-manifold is called a T^2-manifold if it supports an effective T^2 $(= SO(2) \times SO(2))$ action. Given a T^2 action on a 4-manifold M, denote the totality of fixed points and c-orbits (orbits with isotropy group $SO(2)$) by F and C respectively. The topological classification of T^2-manifolds is studied in the following three cases:

(1) $F \cup C = \emptyset$;
(2) $F \neq \emptyset$;
(3) $F = \emptyset, C \neq \emptyset$.

Cases (1) and (2) were treated in [3] and [5]. In this article we will continue this program and study the third case. In Section 2 we give a geometric construction involving the weighted orbit space. Using this construction we are able to compute the fundamental groups, and they in turn determine the homology and cohomology groups of these manifolds. Let M be a T^2-manifold corresponding to the orbit data

$$\{O; g; s; (m_1, n_1), \ldots, (m_s, n_s); (\alpha_1; p_1, q_1; \beta_1), \ldots, (\alpha_t; p_t, q_t; \beta_t)\}$$

in the sense of [2]. Analyzing $\pi_1(M)$, in Section 3 we prove the following:

THEOREM 2. *Let M be the above T^2-manifold. Then the integers $2g + s$, t, $\alpha_1, \ldots, \alpha_t$ and $m = \gcd(m_1, m_2, \ldots, m_s)$ are topologically invariant.*

In Section 4, we apply some elementary surgery theory to compute the second Stiefel–Whitney class of these manifolds. For example:

THEOREM 3. *Let M be a T^2-manifold with orbit invariants*

$$\{O; g; s; (0, 1), (m_2, n_2), \ldots, (m_s, n_s)\}.$$

Then $\omega_2(M) \neq 0$ if and only if there are integers i and j, $2 \leq i, j \leq s$, such that $m_i \equiv m_j \equiv n_i \equiv 1$ (2) and $n_j \equiv 0$ (2).

Unfortunately, $\pi_1(M)$ and $\omega_2(M)$ do not completely classify these manifolds. Some additional results are given in Section 5: If two of the c-orbits have

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