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ONE POINT REGULARITY PROPERTIES OF MULTIPLE FOURIER SERIES WITH GAPS

BY

JOHN D. PESEK, JR.[1]

1. Introduction

We will extend results of Hsieh Xie-Fan [2], G. Freud [1], and M. Izumi, S. Izumi and J.-P. Kahane [3] to several variables and obtain some refinements in one variable. The main result in one variable is that if a Fourier series is lacunary in a certain sense and satisfies a Lipschitz condition of a certain order at some point then the series is in the Lipschitz class of that order.

To establish this and related results, two steps are needed. The first is to use lacunarity at a point to estimate the Fourier coefficients. This is accomplished by a technique due to Noble [7] who worked on a related problem. We modify the technique somewhat and where a special trigonometric polynomial was used, we use a summable function on R^n whose Fourier transform is C^∞ and has compact support. This is a technical improvement especially in several variables. It also makes it possible to investigate two point or n point regularity problems.

The second step is to go from the coefficient estimate to membership in a Lipschitz class, a problem first investigated by Lorentz. We will use a result of Pesek. (See Pesek [8] and [9] for proofs and references.) From these two results and a counting argument we obtain various one point regularity results for multiple Fourier series with gaps. These results apply to certain partial differential equations with constant coefficients. In the last section we give some counterexamples that show our previous results are in some instances best possible.

We wish to point out the possibility of posing analogous problems. Instead of assuming regularity at a point, assume it on larger sets such as neighborhoods, sets of positive measure, or submanifolds of the torus. Then ask what lacunarity conditions will guarantee regularity on the whole torus. For a neighborhood results are known. See Kahane [4]. In this case regularity is expressed in terms of Sobolev spaces.

2. Preliminaries

We summarize the conventions, definitions, and results that we shall need.

Let T^n be the n-dimensional torus. Let R^n be n-dimensional Euclidean space and Z^n be the lattice points of R^n with integer coordinates. By the identification

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