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# RIEMANN-LEBESGUE CENTERS OF PLANE DOMAINS

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## 1. Introduction

Let $G \subseteq \mathbf{C}$ be a plane domain which supports nonconstant bounded analytic functions, and let $\zeta \in G$. For $n = 0, 1, 2, \ldots$ define

$$A_n = A_n(\zeta, G) = \sup \{ | f^{(n)}(\zeta) | : f \in B_H(G), \|f\|_\infty = 1\},$$

where $B_H(G)$ is the space of functions analytic and bounded in $G$, and $\|f\|_\infty$ denotes the supremum norm. A point $\zeta \in G$ such that for each $f \in B_H(G)$,

$$\frac{f^{(n)}(\zeta)}{A_n} \to 0 \quad (n \to \infty),$$

is called a Riemann-Lebesgue center (R.L.C.) of $G$. The name "Riemann-Lebesgue center" is an allusion to the well-known Riemann-Lebesgue lemma, which implies that 0 is a R.L.C. of the unit disc $\{z\colon |z| < 1\}$.

In this paper we shall be concerned with certain problems about R.L.C.'s from the point of view of "hard" analysis. However, the idea of R.L.C. arose in conversation with Stephen D. Fisher with reference to different topologies on function spaces. The latter are dealt with in a paper of Rubel and Ryff [2]. A number of particular topics are considered in greater detail in the papers listed by Rubel and Ryff in their bibliography. The results of the present paper do not, unfortunately, seem to have any applicability to the problem of different topologies on function spaces, so we present them for their intrinsic interest.

## 2. Statement of results

THEOREM 1. *Let* $D = \{z \in \mathbf{C}\colon |z| < 1\}$ *and*

$$A_n = A_n(\zeta; D) = \sup \{|f^{(n)}(\zeta)| : f \in B_H(D), \|f\|_\infty \le 1\},$$

*where* $0 < |\zeta| < 1$. *If* $f \in B_H(D)$, *then*

$$\liminf_{n \to \infty} \frac{|f^{(n)}(\zeta)|}{A_n} = 0;$$

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