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ON THE "STABLE" HOMOTOPY TYPE OF KNOT COMPLEMENTS

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1. Introduction

This paper is concerned with knots of codimension two, that is, embeddings of the (q-2)-sphere S^{q-2} in the q-sphere S^q. By Alexander duality, the *complement* C of the knot (see Paragraph 2) has the same homology groups as the circle S^1, and Levine [7], [8] has proved that if $q \neq 4$, then C is homotopy equivalent to S^1 if and only if the knot is trivial. We will consider those knots for which there is a positive integer n such that $\pi_i C \cong \pi_i S^1$ for $i \leq n$. Thus, all fundamental groups will be infinite cyclic, and all higher homotopy groups will be modules over $\Lambda = \mathbf{Z}[\mathbf{Z}]$, the group ring of the integers. Then, in Theorem 1 (see Paragraph 2), we prove that *$\pi_i C$ is a finitely generated acyclic Λ-module* (see Paragraph 2) *for $n+1 \leq i \leq 2n$* (the "stable range").

Conversely, let X be any space for which $\pi_i X \cong \pi_i S^1$ for $i \leq n$, and $\pi_i X$ is a finitely generated acyclic Λ-module for $n+1 \leq i \leq 2n$. Then, in Theorem 2 (see Paragraph 2), we prove that *there is a knot complement C with the same homotopy type as X through dimension $2n$.*

The first work in this direction was done by Kervaire. In [6] he proved, under the assumptions above, that $\pi_{n+1} C$ is a finitely generated acyclic Λ-module and that any finitely generated acyclic Λ-module can be so realized. In [1], Brown and Dror showed that for $n \geq 2$, the module $\pi_{n+2} C$ has the same characterization as $\pi_{n+1} C$, and that these two modules are independent of one another. In [3], Dror and Dwyer obtained results on homology localizations in the stable range, which imply most of our Theorem 1.

Our Theorems 1 and 2 have analogues for arbitrary *homology circles*, i.e., spaces with the same homology groups as the circle. In Theorems 1′ and 2′ (see Paragraph 2) we show that in the "stable range", the homotopy type of a homology circle has the same characterization as that of a knot complement, except that the acyclic Λ-modules involved are not required to be finitely generated.

Organization of the paper. Paragraph 2 contains the definitions and the statement of our results. Paragraph 3 begins with a review of perfect and acyclic modules, and then proves Theorem 1. Paragraph 4 proves Theorem 2, and the proofs of Theorems 1′ and 2′ are sketched in Paragraph 5.

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