FREE HEEGAARD DIAGRAMS AND EXTENDED NIELSEN TRANSFORMATIONS, II

BY

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Introduction

This is the second of two papers (see also [Cr]) relating a free analogue, called a free splitting homomorphism, of the Stallings–Jaco formalism for Heegaard splittings of 3-manifolds (see [St] and [Jc]) with the theory of extended Nielsen transformations. Free splitting homomorphisms are homorphisms of the form

$$\psi = \psi_1 \times \psi_2 \colon G^m \to X^n \times Y^n$$

where G^m, X^n, and Y^n are free groups of ranks m, n, and n respectively and each of the factor homomorphisms $\psi_1 \colon G^m \to X^n$ and $\psi_2 \colon G^m \to Y^n$ is surjective. A theory of equivalence and stable equivalence for free splitting homomorphisms is developed in [Cr] modelled on the corresponding theory for Heegaard diagrams. Extended Nielsen transformations may be regarded as certain special Tietze transformations that preserve the deficiency of a group presentation. They will be reviewed in §1. Sometimes extended Nielsen transformations include operations that allow one to add new generators and relations to a presentation—i.e. to stabilize the presentation—and sometimes these operations are excluded.

A connection between the free splitting homomorphism theory and the theory of extended Nielsen transformations is made in [Cr] by normalizing a free splitting homomorphism $\psi = \psi_1 \times \psi_2 \colon G^m \to X^n \times Y^n$ so that for free bases $\{g_i \colon i \le m\}$ for G^m and $\{x_i \colon i \le n\}$ for X^n we have $\psi_1(g_i) = x_i$ $(i \le n)$ and $\psi_1(g_{i+n}) = 1$ $(i \le m - n)$. In a manner analogous to the reading of a group presentation for the fundamental group of a 3-manifold from a Heegaard diagram for the 3-manifold, we associate a group presentation $\mathscr{P}(\psi) = \langle Y^n \colon (r_i) \rangle$ with the normalized ψ where (r_i) is the $(m - n)$-tuple $(\psi_2(g_{i+n}))$. Theorem 4.1 of [Cr] says that two free splitting homomorphisms ψ and ϕ in normal form and with associated group presentations $\mathscr{P}(\psi)$ and $\mathscr{P}(\phi)$ are stably equivalent if and only if the presentations $\mathscr{P}(\psi)$ and $\mathscr{P}(\phi)$ are equivalent under extended Nielsen transformations including stabilization.

Our interest in this paper will be in establishing a more detailed connection between the free splitting homomorphism theory and the theory of extended Nielsen transformations and in using the information thus obtained to gain further insight into the difficult Andrews–Curtis conjecture