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ASYMPTOTIC EXPANSIONS FOR THE COMPACT QUOTIENTS OF PROPERLY DISCONTINUOUS GROUP ACTIONS

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1 Introduction

Let M be a connected Riemannian manifold and Γ a group acting isometrically, effectively, and properly discontinuously on M with compact quotient space $\bar{M}=\Gamma\backslash M$. The orbit space $\bar{M}$ is not necessarily a manifold. Suppose that $\pi: M \to \bar{M}$ denotes the associated projection. A function f defined on $\bar{M}$ is said to be of class $C^l(\bar{M})$ if $f \circ \pi \in C^l(M)$. Since Γ acts isometrically, the Laplacian Δ of M is Γ-invariant and Δ induces an operator $\bar{\Delta}$ on $C^2(\bar{M})$.

The Laplacian $\bar{\Delta}$ has a self-adjoint extension to $L^2(\bar{M})$ with pure point spectrum $\lambda_1 \leq \lambda_2 \leq \cdots | \uparrow \infty$. Our main result, Theorem 4.8, is the asymptotic formula as $t \downarrow 0$:

$$\sum_{i=1}^{\infty} e^{-t\lambda_i} \sim (4\pi t)^{-n/2} \sum_{i=0}^{\infty} a_i t^i$$

where $n=\dim(M)$. Here $a_0=\operatorname{vol}(\bar{M})$, the volume of $\bar{M}$. The higher order terms a_i may be computed by the method of the author's earlier paper [5].

If $M=G/K$, a symmetric space, and $\Gamma \subset G$ is as above, then the first term of our asymptotic formula was obtained by N. Wallach [13]:

$$\sum_{i=1}^{\infty} e^{-t\lambda_i} \sim (4\pi t)^{-n/2} \operatorname{vol}(\bar{M}).$$

His method relied on the algebraic fact that for symmetric spaces the Γ-action may be factored through a finite group action on a compact Riemannian manifold [3]. As shown below, this technique is not available in the general case, when M need not be symmetric.

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The results of this paper generalize easily to the Laplacian with coefficients in a bundle.

2 Properly discontinuous actions

Let M be a connected manifold and Γ a group acting differentiably and properly discontinuously on M with compact quotient $\bar{M}=\Gamma\backslash M$. Recall that

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