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THE LOCALLY FREE CLASSGROUP OF THE SYMMETRIC GROUP

BY

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1. Introduction and statement of results

Let S_n denote the symmetric group on n letters, and let $\mathrm{Cl}(\mathbf{Z}S_n)$ be the classgroup of finitely generated locally free $\mathbf{Z}S_n$ modules. $\mathrm{Cl}(\mathbf{Z}S_n)$ has been studied in [5], [6] and [11]. (See also [12]). By use of two sets of homomorphisms on $\mathrm{Cl}(\mathbf{Z}S_n)$ namely those developed by A. Fröhlich in [1], and those developed by the author in [9], we are able to describe $\mathrm{Cl}(\mathbf{Z}S_n)$, up to a two group, in terms of two groups of polynomials.

I should like to express my thanks to S. Ullom who originally suggested to me that the representation theory of the symmetric group might be particularly applicable to the calculation of the locally free classgroup.

Let $\mathbf{Z}$ denote the ring of rational integers and let $\mathbf{Q}$ be the field of rationals. If l is a prime of $\mathbf{Z}$, we define $\mathbf{Z}_l$ to be the ring of l-adic integers and $\mathbf{Q}_l$ to be the rational l-adic field. If Γ is a finite group we let R_Γ be the ring of virtual characters of Γ. For any ring R we denote the group of units of R by R^*.

Let $\Lambda_n^{(m)}$ be the additive group of symmetric polynomials of degree n over $\mathbf{Z}$, in the m variables $x_1, x_2 \cdots x_m$.

We have a homomorphism of groups $\Lambda_n^{(m+1)} \to \Lambda_n^{(m)}$ given by setting $x_{m+1} = 0$. We let $\Lambda_n = \lim_{\leftarrow} \Lambda_n^{(m)}$ (the limit being taken with respect to the above projective system). For each rational prime l we set $\Lambda_{n,l} = \mathbf{Z}_l \otimes_{\mathbf{Z}} \Lambda_n$.

In the usual way we identify the conjugacy classes of S_n with the partitions of n (via cycle structure). If π is a partition of n then $|\pi|$ denotes the number of elements in the conjugacy class π. For $a \in \mathbf{Z}$, π^a denotes that conjugacy class to which the ath powers of elements of π belong.

If π is a partition of n, $n = r_1 + \cdots + r_k$, then we define the symmetric polynomial $\sigma_\pi^{(m)} \in \Lambda_n^{(m)}$ by setting

$$\sigma_\pi^{(m)} = \prod_{i=1}^{k} (x_1^{r_i} + \cdots x_m^{r_i}).$$

We set $\sigma_\pi = \lim_{\leftarrow} \sigma_\pi^{(m)}$.

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