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# ON A CONNECTIVITY PROPERTY INDUCED BY THE $\theta$-CLOSURE OPERATOR

BY

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**In memory of Professor W. W. S. Claytor**

## 1. Introduction

The notion of $\theta$-closure of a subset $A$, $\mathrm{cl}_\theta(A)$, of a topological space was introduced by Veličko for the purpose of studying the important class of $H$-closed spaces in terms of arbitrary filterbases [23] and in order to generalize the Taimanov Extension Theorem [24]. Recently, the operator $\mathrm{cl}_\theta$ has been studied by a number of researchers (See [1], [2], [4], [7], [8]–[10], [11]–[15].) Herrington and Long [10] have characterized minimal Hausdorff and $C$-compact spaces in terms of this operator. Dickman and Porter [1] have utilized the operator to show that while $H$-closed spaces are not necessarily of the second category, these spaces do satisfy a property of "second category type"; they also used $\mathrm{cl}_\theta$ to give a characterization of those Hausdorff spaces in which the Fomin $H$-closed extension operator commutes with the projective cover (absolute) operator [1]; these same authors used the operator in [2] to study the extension function problem for the $\theta$-continuous functions of Fomin [5] between Hausdorff spaces. It has also been shown recently [11] that $\mathrm{cl}_\theta(A)$ is quasi $H$-closed relative to an $H$-closed space in the sense of Porter and Thomas [21] for each subset $A$ of the space. In this paper, we use $\mathrm{cl}_\theta$ to introduce a collection of subsets of a space which we call $\theta$-connected relative to the space, and we study this collection of subsets. This collection of subsets of a space always contains the collection of connected subsets of the space but these two classes do not always coincide. All spaces are assumed to be Hausdorff.

In Section 2, we provide some elementary properties of subsets which are $\theta$-connected relative to a space and use these subsets to generalize the well-known theorem that the intersection of continua directed by inclusion is a continuum. Noiri [20] has established that if a space $X$ is connected and the function $g: X \to Y$ is a surjection which is weakly-continuous in the sense of Levine, then $Y$ is connected. It is interesting to find that if $g$ is weakly-continuous, then $g(A)$ is $\theta$-connected relative to $Y$ for each connected $A \subset X$. Another of the results in this section is a parallel of the well-known Wallace Theorem. This parallel is provided for the $\theta$-rigid subsets of Dickman and

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