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COMPLEMENTS OF CODIMENSION-TWO SUBMANIFOLDS II-HOMOLOGY BELOW THE MIDDLE DIMENSION

BY

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Introduction

This paper studies the modules that can occur as the homology modules below the middle dimension of the complement of a codimension-two imbedding of compact manifolds and it therefore forms a continuation of [26]. Particular attention is focused upon the modules that can occur as the homology modules of a certain covering space of the complement which, in the case of knots, is the infinite cyclic cover.

The only case of this problem that has been studied before is that of high-dimensional knots. In [19], Kervaire characterized the *first nonvanishing* homology module of a knot complement when its fundamental group is $\mathbf{Z}$. This work was continued by Levine in a series of papers that culminated in [20] in which he obtained a complete and simultaneous characterization of the homology of the infinite cyclic cover of a knot complement, except for a slight difficulty in dimension two. The present work studies classes of imbeddings, known as realizations of Poincaré imbeddings (these are defined in Section 1), that include high-dimensional knots as well as other well-known classes of imbeddings such as local knots and knotted lens spaces and obtains complete characterizations, in many cases, of the homology of the complement below the middle dimension. The results of this paper apply equally to smooth, *PL*, and topological imbeddings and manifolds.

In constructing imbeddings with prescribed homology modules in the complement, an algebraic K-theoretic obstruction is encountered, called the χ-invariant in this paper, that takes its value in a *relative* algebraic K-group and which incorporates aspects of both the Wall finiteness obstruction and Whitehead torsion.

In Section 2, where this invariant is discussed, it is shown that all elements of the relative algebraic K-group $K'_0(f)$ (see [4, Chapter 9]) occur as χ-invariants of suitable chain complexes so that we get a *geometric* interpretation of $K'_0(f)$.

In a special case that occurs in the study of knotted lens spaces, this invariant is explicitly calculated; it is shown that, in this case, it can be interpreted as an alternating product of "Alexander polynomials" of complementary homology modules evaluated at a primitive root of unity. Specifically, our result is:

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