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INJECTIVE BP_*BP-COMODULES AND LOCALIZATIONS OF BROWN-PETERSON HOMOLOGY

BY

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1. Introduction

BP is the Brown-Peterson spectrum for a fixed prime p; its homotopy is

$$BP_* \cong \mathbf{Z}_{(p)}[v_1, v_2, \ldots].$$

By convention, $v_0 = p$. $BP_*X = \pi_*(BP \wedge X)$ is a comodule over $BP_*BP \cong BP_*[t_1, t_2, \ldots]$. Let $\mathscr{BP}$ be the category of all BP_*BP-comodules and comodule maps. The only prime ideals of BP_* which are in $\mathscr{BP}$ are

$$I_0 = (0), I_1 = (p), \ldots, I_n = (p, v_1, \ldots, v_{n-1}), \ldots,$$

and

$$I_\infty = \bigcup_n I_n = (p, v_1, v_2, \ldots).$$

The Hurewicz homomorphism gives a right unit $\eta_R: BP_* \to BP_*BP$ and $\eta_R(v_n) \equiv v_n$ modulo $I_n BP_*BP$. (N.B. $\eta_R(v_1) = v_1 + pt_1 \neq v_1$.)

We say that a BP_*-module M is $\mathscr{BP}$-injective if $\mathrm{Ext}^i_{BP_*}(A, M) = 0$ for all $i > 0$ and all comodules A in $\mathscr{BP}$. We define the $\mathscr{BP}$-weak dimension of M, $\mathrm{w.dim}_{\mathscr{BP}} M$, to be less than $n + 1$ if $\mathrm{Tor}_j^{BP_*}(A, M) = 0$ for all $j > n$ and all comodules A in $\mathscr{BP}$. If M, itself, is a connected comodule in $\mathscr{BP}$, $\mathrm{w.dim}_{\mathscr{BP}} M$ is the same as the BP_*-projective dimension of M [8]. Our main algebraic result can be considered to be the dual of Landweber's exact functor theorem [8].

THEOREM 1.1. *For a BP_*-module M to be $\mathscr{BP}$-injective, it suffices that it satisfy two conditions:*

(i) *For each integer $n \geq 0$, $\mathrm{Hom}_{BP_*}(BP_*/I_n, M)$ is v_n-divisible.*
(ii) $\mathrm{w.dim}_{\mathscr{BP}} M < \infty$.

Miller, Ravenel, and Wilson [13] develop a "chromatic resolution" of

$$BP_*: 0 \to BP_* \to M^0 \to M^1 \to \cdots.$$

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