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EMBEDDINGS OF $S^n \times M$ IN $S^{n+2} \times M$ FORM A GROUP

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Introduction

This paper describes group structures for a large class of codimension two embedding problems. The classic example of algebraic structure in an embedding problem is furnished by the knot cobordism groups of [9], [11], [13]. Our general study uses homology surgery theory, first developed and applied to the codimension two placement problem in [7].

Let M be an arbitrary k-dimensional compact manifold. This paper classifies standard M-knots, i.e., embeddings $f\colon S^n \times M \to S^{n+2} \times M$ which are homotopic, rel boundary, to the standard inclusion. Using a definition of cobordism based on concordance of embeddings, we prove that the set $G_n^t(M)$ of cobordism classes of such M-knots forms an abelian group in a natural way, provided $n \geq 2$ and $n + k \geq 4$. This was known previously for M simply connected [7] and for a certain class of non simply connected M [16]. Herein we treat the general case by devising a variant of surgery theory which studies the normal cobordism problem for simply split simple homotopy equivalences [5], [8]. The desired group structure is obtained by exhibiting $G_n^t(M)$ as a subgroup of a relative homology surgery group in this theory. For all M, we interpret this group structure geometrically. When M is a point, $G_n^t(M)$ coincides with the knot cobordism groups of [11], [13], wherein the group operation is defined by taking connected sum of knots.

Two embeddings $f, g\colon S^n \times M \to S^{n+2} \times M$ are called cobordant if f is concordant to $\phi f \psi$, where ϕ and ψ are certain allowable automorphisms of $S^{n+2} \times M$ and $S^n \times M$ respectively. The set of cobordism equivalence classes is denoted $G_n^t(M)$; see Section 1 for a precise definition, as well as the reason for including the superscript "t" in the notation. Our results are valid for M a smooth (resp. piecewise linear, topological) manifold, provided we restrict attention to smooth (resp. piecewise linear locally flat, topological locally flat) embeddings and concordances, and require ϕ and ψ to be diffeomorphisms (resp. piecewise linear homeomorphisms, homeomorphisms). For simplicity, discussions and results are stated for the smooth case.

The groups $G_n^t(M)$ do not, in general, satisfy the fourfold periodicity proved

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