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DETERMINING SETS FOR MEASURES ON $\mathbf{R}^n$

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1. Introduction

Let M be a class of measures on $\mathbf{R}^n$. A Borel set E is said to be a determining set for M if $\mu, \nu \in M$, and $\mu(x + E) = \nu(x + E)$ for all $x \in \mathbf{R}^n$ implies $\mu = \nu$.

Let $\Gamma = \{x = (x_1, \ldots, x_n) \in \mathbf{R}^n;\ x_i \geq 0 \text{ for all } i\}$. Then it is well known to probabilists that Γ is a determining set for the class P of all probability measures on $\mathbf{R}^n$. (For a Fourier transform theoretic proof of this, for $n = 2$, see [4].)The aim of this paper is to generalize the above result to an arbitrary Borel set E of positive Lebesgue measure contained in Γ (see Theorem 3.3). The proof of this theorem is based on Proposition 3.1 which is very similar to the results in [4]. (For a discussion of determining sets in the context of locally compact abelian groups or symmetric spaces see [2].)

2. Notation and terminology

For any unexplained notation or terminology please see [3].

Throughout this paper λ denotes the Lebesgue measure on $\mathbf{R}^n$. Let C denote the class of all (finite) complex measures and P the class of all probability measures on $\mathbf{R}^n$. If T is a tempered distribution (in the sense of Schwartz), then $\hat{T}$ denotes the Fourier transform of T (which is again a distribution) and Supp T denotes the (closed) support of T. For standard facts regarding distributions, Fourier transforms etc., see [3]. If g is a bounded Borel function on $\mathbf{R}^n$, then g defines a tempered distribution (see [3]) and $\hat{g}$ will denote the (distributional) Fourier transform of g. If μ is a finite complex measure, then $\mu * g$ is the bounded Borel function defined by

$$(\mu * g)(x) = \int_{\mathbf{R}^n} g(x - y)\, d\mu(y)$$

Finally, we note that for a complex measure or an L^1-function the usual notion of Fourier transform coincides with the notion of distributional Fourier transform.

If M is a class of measures on $\mathbf{R}^n$, a Borel set E of $\mathbf{R}^n$ is said to be a "determining set" for M if $\mu, \nu \in M$ and $\mu(x + E) = \nu(x + E)$ for all $x \in \mathbf{R}^n$ implies $\mu = \nu$.

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