

EXTENSIONS OF CONTINUOUS FUNCTIONS ON DENSE SEMIGROUPS

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1. Introduction

Let S be a dense subsemigroup of a semitopological semigroup T . In this paper we consider the following "extension problem": Given certain subalgebras A of $C(S)$ and B of $C(T)$ (say the algebras of weakly almost periodic functions), determine minimal conditions on S and T such that every function in A extends to a member of B ; in symbols, $A \subset B|_S$.

A number of interesting results pertaining to this problem have appeared in the literature in recent years. For example, A. T. Lau, generalizing a result of S. J. Wiley [19], has shown that if T is a topological group, then $UC(S) \subset C(T)|_S$ [10]. P. Milnes, improving on Lau's result, showed that if T is a topological semigroup which is a group then $LMC(S) \subset C(T)|_S$ and, consequently, $AP(S) = AP(T)|_S$ and $WAP(S) = WAP(T)|_S$ [13]. The proofs of these results depend critically on both the group structure of T and the joint continuity of multiplication. In this paper we generalize and complement these results, requiring that multiplication in T be only separately continuous and that T satisfy some condition generally weaker than the group property. Some examples of the type of results we obtain are the following:

(A) If T is topologically right simple and contains a right identity, then $LUC(S) \subset C(T)|_S$.

(B) If T is topologically left and right simple (for example, if T is a semitopological group), then $WAP(S) \subset UC(S) \subset C(T)|_S$ and hence $WAP(S) = WAP(T)|_S$ and $AP(S) = AP(T)|_S$.

(C) If T is topologically simple, then $SAP(S) = SAP(T)|_S$.

The central theme of this paper is that of right topological compactification of a semigroup, and this notion is used systematically in the proofs of our theorems. Although we do not do so, many of our results may also be phrased in terms of these compactifications. For example, (C) may be restated as follows: If T is topologically simple, then S and T have the same SAP compactification (up to isomorphism).

Received June 9, 1981.