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ADDENDUM TO MY PAPER "ON COLORING MANIFOLDS"

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An important paper by Grünbaum [1], which had escaped my attention until now, contains the following theorem: If $m \geqslant 2$ then one can assign $6(m-1)$ colors to the $(m-2)$-simplices of any simplicial complex imbedded in $\mathbf{R}^m$ in such a way that any two $(m-2)$-simplices incident to the same $(m-1)$-simplex have different colors. A fortiori, this implies the finiteness of the numbers $\mathrm{ch}_{m-2}(S^m)$ of [2].

It is easily seen that Theorems 1 and 2 of [2] are equivalent to the following.

THEOREM A. *If X is any closed m-dimensional pseudomanifold ($m \geqslant 2$), then*

$$\mathrm{ch}_{m-2}(X) \leqslant \left\{\frac{m(m+1)}{m-1}[1 + b_{m-1}(X; \mathbf{Z}_2)]\right\}.$$

Further if K is any subcomplex of a triangulation of X and contains at least one $(m-2)$-simplex, then

$$\frac{m-1}{m+1}\alpha_{m-1}(K) \leqslant \alpha_{m-2}(K) + b_{m-1}(X; \mathbf{Z}_2) - 1.$$

We will now use the ideas of Grünbaum [1] to show that this theorem can be significantly improved when the hypotheses are strengthened somewhat.

THEOREM B. *If X is any closed triangulable manifold ($m \geqslant 3$), then $\mathrm{ch}_{m-2}(X) \leqslant 6$. Further if K is any subcomplex of a triangulation of X and contains at least one $(m-2)$-simplex, then $m\alpha_{m-1}(K) < 6\alpha_{m-2}(K)$.*

Proof. The first part will follow from the second (as in the proof of Theorem 2 of [2], for example). Let K be a subcomplex of a triangulation L of X and let $\sigma_1, \sigma_2, \ldots, \sigma_t$ be the $(m-3)$-simplices of K which are incident to at least one $(m-2)$-simplex of K. Since X is an m-manifold ($m \geqslant 3$), $Lk_1\sigma_i$, $1 \leqslant i \leqslant t$, is a triangulation of the 2-sphere S^2. Further $Lk_K\sigma_i$, $1 \leqslant i \leqslant t$, is a subcomplex of $Lk_L\sigma_i$ and contains at least one vertex.

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