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FACTORIZATION OF POSITIVE MULTILINEAR MAPS

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1. Introduction

Let (X, μ) be a finite measure space and let $L_0(X, \mu)$ denote the space of (equivalence classes of) all μ-measurable functions on X. E. M. Nikisin [6] and B. Maurey [5] proved several factorization theorems for linear and sub-linear operators, where a (sub-)linear operator T from a Banach space E into $L_p(X, \mu)$ $(p \geq 0)$ factors through (weak-) L_r $(r \geq 1)$, if there exists $\phi \in L_s$ for some $s \geq 0$ with $\phi > 0$ a.e., such that

$$\frac{1}{\phi} \cdot T(E) \subseteq (\text{weak-})L_r.$$

For an excellent survey of these theorems and the many applications of them we refer to J. E. Gilbert's paper [2]. In this same article Gilbert indicates that there are available versions of weak-type factorizations for maximal operators defined by multilinear operators, but it was also noticed that strong-type factorizations for multilinear operators had not yet been studied. In this paper we shall prove strong-type factorizations for positive multilinear operators. Our approach uses the positive projective tensor product of Banach lattices and we also use some of the linear operator results of Nikisin and Maurey. The results for bilinear operators are typical for the multilinear case, but we could not restrict ourselves to the bilinear case. To prove Theorem 3.2 and Theorem 3.5 for bilinear operators with values in L_r with $r > 0$, we need the result of the same theorems for trilinear operators. Therefore we consider the general multilinear case. For the same reason we shall consider tensor products of n Banach lattices. The organization of this paper is as follows. In Section 2 we develop the necessary machinery of the theory of tensor products of Banach lattices. In Section 3 we prove the factorization theorems for positive multilinear operators from $L_{p_1} \times \cdots \times L_{p_n} \to L_q$ where $q \geq 0$.

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