

STRUCTURE OF FOLIATIONS ON 2-MANIFOLDS

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Introduction

In this paper, we intend to study qualitative properties of foliations with finitely many singularities on closed 2-manifolds. Considering such a foliation as a regular foliation on the punctured 2-manifold obtained from a closed 2-manifold by removing the singular points, we will give an analogy of a structure's theorem (in Salhi [10], Theorem 1) on codimension one regular foliations on closed manifolds. Singular foliations on 2-manifolds have been investigated by many authors from a geometric point of view (for example, see [2], [5], [6]). We are interested more precisely in the following questions:

1. Describe the foliation near a leaf.
2. Establish a structure's theorem.

We mention that the results given here are known for foliations with singularities saddles and/or thorns.

In Section 1, we give some preliminaries (definitions and notations of the general theory of singular foliations on 2-manifolds, and some topological results which will be needed later.). In Section 2, we give a description of foliations near a leaf, especially near an exceptional leaf, by establishing analogues of Sacksteder's Theorem [9] for singular foliations on 2-manifold (Theorems 2.1 and 2.2). Some consequences as in [10], [11] are given.

1. Preliminary

(A) Basic definitions.

This section is devoted to the basic facts of the general theory of singular foliations on 2-manifolds. Let $\mathcal{F}$ be a C^∞ singular foliation with a finite number of singularities on a compact orientable 2-manifold S of genus g . We let $\text{sing } \mathcal{F}$ be the set of singularities of $\mathcal{F}$, $\mathcal{F}/U$ the restriction of $\mathcal{F}$ to an invariant open set U of S , $\mathcal{F}^*$ the restriction of $\mathcal{F}$ to $S^* = S - \text{sing } \mathcal{F}$, and let U_1 be the complement in S^* of the union of closed leaves of $\mathcal{F}^*$. By [3], Theorem p. 386, U_1 is an open invariant set of S .

Received April 7, 1997.

1991 Mathematics Subject Classification. Primary 57R30; Secondary 58F18.

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