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EXTREME POINTS OF UNIT BALLS OF QUOTIENTS OF L^∞ BY DOUGLAS ALGEBRAS

BY

KEIJI IZUCHI

1. Introduction

Let H^∞ be the space of bounded analytic functions in the unit disk D. Identifying with boundary functions, we consider H^∞ as an (essentially) uniformly closed subalgebra of L^∞, the space of bounded measurable functions on the unit circle ∂D with respect to the normalized Lebesgue measure m. Every uniformly closed subalgebra between H^∞ and L^∞ is called a Douglas algebra. In this paper, B always denotes a Douglas algebra. It is well known that $H^\infty + C$ is the smallest Douglas algebra containing H^∞ properly, where C is the space of continuous functions on ∂D. The reader is referred to [5] and [12] for the theory of Douglas algebras, and [4] for uniform algebras.

In this paper, we will study the following problem.

PROBLEM. *For which Douglas algebra B, does* $\mathrm{ball}(L^\infty/B)$ *have extreme points?*

We denote by $\mathrm{ball}(Y)$ the closed unit ball of a Banach space Y. A point x in $\mathrm{ball}(Y)$ is called extreme if $x = (x_1 + x_2)/2$ for x_1, x_2 in $\mathrm{ball}(Y)$ implies $x = x_1 = x_2$. An equivalent condition for a point x in $\mathrm{ball}(Y)$ to be extreme is that the condition $\|x \pm y\| \le 1$, $y \in Y$, implies $y = 0$.

Up to now, we know the following theorems about extreme points of $\mathrm{ball}(L^\infty/B)$.

KOOSIS' THEOREM [9]. $\mathrm{ball}(L^\infty/H^\infty)$ *has an extreme point. A point $f + H^\infty$ in* $\mathrm{ball}(L^\infty/H^\infty)$ *is an extreme point if and only if there is a function h in $f + H^\infty$ such that $|h| = 1$* a.e. *dm and $\|h + g\| > 1$ for every $g \in H^\infty$ with $g \ne 0$.*

AXLER, BERG, JEWELL AND SHIELDS' THEOREM [2]. $\mathrm{ball}(L^\infty/H^\infty + C)$ *does not have extreme points.*

For a subset F of ∂D, we denote by L_F^∞ the space of functions in L^∞ which can be redefined on a set of measure zero so as to become continuous at every

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