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REAL PARTS OF NORMAL EXTENSIONS OF SUBNORMAL OPERATORS[1]

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1. Introduction and main theorem

A bounded linear operator S on a separable Hilbert space H is said to be subnormal if S has a normal extension N to a Hilbert space $K \supset H$. In case S has no normal part then S is said to be a pure subnormal operator. Further, N is called the (essentially unique) minimal normal extension if the only reducing space of N which contains H is K. (For the basic properties of subnormal operators, see Halmos [3], Chapter 21, and for a detailed exposition of the subject, see Conway [2].) Since H is invariant under N then $H^{\perp} = K \ominus H$ is invariant under N^*. As in Conway [1], the operator $T = N^*|H^{\perp}$, is called the dual of $S = N|H$. Further, one can express N and N^* as operator matrices

$$(1.1)\quad N = \begin{bmatrix} S & X \\ 0 & T^* \end{bmatrix} \quad\text{and}\quad N^* = \begin{bmatrix} S^* & 0 \\ X^* & T \end{bmatrix} \quad\text{on}\quad K = H \oplus H^{\perp}.$$

In Olin [6], p. 228, it is shown that since S is pure with minimal normal extension N then T is also pure with minimal normal extension N^*. Further ([1], p. 196), T is the dual of S with spectrum $\sigma(T) = \{\bar{z} : z \in \sigma(S)\}$. Simple calculations with the matrices of (1.1) show that

$$(1.2)\qquad S^*S - SS^* = XX^*, \quad T^*T - TT^* = X^*X$$

and

$$(1.3)\quad \mathrm{Re}(N) = \tfrac{1}{2}(N + N^*) = \begin{bmatrix} \mathrm{Re}(S) & \tfrac{1}{2}X \\ \tfrac{1}{2}X^* & \mathrm{Re}(T) \end{bmatrix} \quad\text{on}\quad K = H \oplus H^{\perp}.$$

Since S and T are pure subnormal (hence also hyponormal) operators, both $\mathrm{Re}(S)$ and $\mathrm{Re}(T)$ are absolutely continuous operators on H and $H^{\perp}$, respectively; Putnam [8], pp. 42–43.

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