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TANGENTIAL LIMITS OF BLASCHKE PRODUCTS AND FUNCTIONS OF BOUNDED MEAN OSCILLATION

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1. Introduction

Let Δ and C denote the disk $\{|z| < 1\}$ and its boundary $\{|z| = 1\}$. For $\{a_k\}$ a sequence in Δ satisfying the Blaschke condition $\Sigma(1 - |a_k|) < \infty$, let $B(z) = B(z, \{a_k\})$ denote the Blaschke product

$$\prod \frac{\bar{a}_k}{|a_k|} \frac{a_k - z}{1 - \bar{a}_k z}, \quad z \in \Delta,$$

where we set $\bar{a}_k/|a_k| = -1$ if $a_k = 0$. Let H^p, $0 < p \leq \infty$, denote the usual Hardy classes of analytic functions on Δ, and let BMOA be the space of analytic functions on Δ having bounded mean oscillation. Corresponding to each Blaschke product $B(z)$, let

$$K_*(B) = K_2(B) \cap \text{BMOA},$$

where

$$K_2(B) = H^2 \ominus BH^2$$

is the orthogonal complement in H^2 of the invariant subspace BH^2. Recall that every function in H^p, $0 < p \leq \infty$, has finite nontangential limits defined almost everywhere (a.e.) with respect to linear Lebesgue measure $(d\theta)$ in C. Also, every Blaschke product is contained in H^∞ and has nontangential limits of modulus 1 a.e. $[d\theta]$, and $H^\infty \subsetneqq \text{BMOA} \subsetneqq \bigcap_{0 < p < \infty} H^p$. (See [7] and [9] for background concerning the spaces of functions defined above.)

In this paper we give conditions on the zero sequence $\{a_k\}$ of the Blaschke product $B(z, \{a_k\})$ which insure the existence of certain nontangential and tangential limits for every one of its subproducts and for the functions in the class $K_*(B)$. The following notation will be used to state our main results and

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