ILLINOIS JOURNAL OF MATHEMATICS
Volume 32 Number 3, Fall 1988

WILD CATEGORIES OF PERIODIC MODULES

BY

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Dedicated to the memory of Irving Reiner, friend, colleague and teacher

Let K be a field of characteristic $p > 0$ and $G = \langle x, y\rangle$ be an elementary abelian group of order p^2. It has long been known (Heller-Reiner [2]), that if $p > 2$ the category of left KG-modules is wild. Basically this means that there is no possibility of classifying the indecomposable objects in the category. However, there seems to be a general misconception that suitable subcategories such as the full subcategory of periodic modules, should be better behaved. The purpose of this note is to demonstrate that such is not the case. Indeed we show that, generally, the full subcategory of all KG-modules whose cohomology rings are annihilated by a fixed non-nilpotent element ζ of $H^2(G, K)$ is a wild category. We will not belabor the point by proving it in every possible case. For simplicity, only the case in which $p \geq 7$ and ζ is the Bockstein of a nonzero element of $H^1(G, F_p)$ will be considered. We hope that the reader will regard this example as sufficient.

Considering the terminology and notation of varieties and cohomology rings we refer the reader to [1].

This work was completed while the first author was visiting the University of Sao Paulo. He would like to thank the USP for their hospitality and the International Programs Division of the NSF for its financial support.

For convenience let $X = x - 1$ and $Y = y - 1$. Then a KG-module may be considered as a $K[X, Y]$-module in which $X^pM = 0$ and $Y^pM = 0$. Let $\zeta \in H^2(G, k)$ be the Bockstein of the element $\eta \in H^1(G, K)$ represented by η: $\Omega(K) \to K$, where $\eta(X) = 1$ and $\eta(Y) = 0$, and $\Omega(K)$ is the kernel of the augmentation $KG \to K$.

LEMMA 1. *Let M be a KG-module such that $M_{\langle x\rangle}$ is a free $K\langle x\rangle$-module and*

$$Y^{(p-1)/2}M = 0.$$

Then ζ annihilates $\operatorname{Ext}^*_{KG}(M, M)$.

Received September 29, 1987.

[1]Partially supported by grants from the National Science Foundation.