ILLINOIS JOURNAL OF MATHEMATICS
Volume 32, Number 3, Fall 1988

MOODY'S INDUCTION THEOREM[1]

BY

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Dedicated to the Memory of Irving Reiner

1. Introduction

Our purpose is to give a proof of the recent remarkable induction theorem of John Moody [1], a proof that is straightforward and more or less self contained. Let Γ be a finitely generated abelian by finite group, and let $S * \Gamma$ be a crossed product of a left noetherian ring S with Γ. Let $G_0(S * \Gamma)$ denote the Grothendieck group of the category of all finitely generated $S * \Gamma$-modules. For any subgroup F of Γ, there is a map $G_0(S * F) \to G_0(S * \Gamma)$ given by sending the class $[M]$ of an $S * F$-module M to the class $[S * \Gamma \otimes_{S * F} M]$ of the induced module.

MOODY'S THEOREM. *Let α be the sum of the maps from $\Sigma G_0(S * F)$ to $G_0(S * \Gamma)$, where F varies over all finite subgroups of Γ. Then α is surjective.*

As an application to G_0 of group rings, let H be a polycyclic by finite group, and let k be a noetherian ring.

MOODY'S THEOREM FOR POLYCYCLIC BY FINITE GROUPS. *The map from $\Sigma G_0(kF)$ to $G_0(kH)$, given by the sum of inductions from finite subgroups F of H, is surjective.*

To prove this, let H_1 be a normal subgroup of H of smaller Hirsch length than H, such that $H/H_1 = \Gamma$ is abelian by finite, and write the group ring kH as a crossed product $(kH_1) * (H/H_1)$. Then use induction on the Hirsch length.

Here is an outline of our proof of Moody's Theorem. Let A be a finitely generated free abelian normal subgroup of Γ of finite index, and let G denote

Received September 2, 1987.

[1]Research supported in part by NSERC of Canada.