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MORDELL-WEIL GROUPS AND THE GALOIS MODULE STRUCTURE OF RINGS OF INTEGERS

BY

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This paper is dedicated to the memory of Irv Reiner in recognition of much kindness shown

1. Introduction and statement of results

In [5], A. Fröhlich introduced the notion of a Kummer order—taken with respect to the multiplicative group law. The Galois module structure (at rational level) of such orders was then determined by the author in [20]. The purpose of this article is to introduce and study the corresponding Galois module properties of analogous orders, when the multiplicative group law is replaced by the group law of an abelian variety.

Before stating our results, we first fix some notation. Let $\mathbf{Q}^c$ denote the algebraic closure of $\mathbf{Q}$, which we view as embedded in $\mathbf{C}$ once and for all; we write ρ for the complex conjugation automorphism of $\mathbf{C}$; given a number field $L \subseteq \mathbf{Q}^c$, we put $\Omega_L = \mathrm{Gal}(\mathbf{Q}^c/L)$.

We let K denote a CM number field, that is to say K is a totally imaginary extension of a totally real field. We then fix a CM type Φ of K; thus $\Phi = \{\varphi_1, \dots \varphi_n\}$, is a transversal, modulo the action of ρ, of the set of field embeddings from K into $\mathbf{Q}^c$. We write K' for the field generated by all elements of the form $\sum_i x^{\varphi_i}$ for $x \in K$: we call K' the reflex field of K with respect to Φ. In the sequel we shall always suppose the CM type (K, Φ) to be simple, that is to say K identifies with the reflex of K'. We write N_Φ: $K^* \to K'^*$ for the reflex norm $N_\Phi(x) = \prod_i x^{\varphi_i}$, and we shall also write N_Φ for the corresponding norm map on idèles and ideals.

Let A denote an Abelian variety of dimension $n = \frac{1}{2}[K:\mathbf{Q}]$ which admits complex multiplication by $\mathfrak{O}_K$. For points P, Q on A we write $P +_A Q$ for

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