ILLINOIS JOURNAL OF MATHEMATICS
Volume 32, Number 3, Fall 1988

A MAYER-VIETORIS SEQUENCE FOR PICARD GROUPS, WITH APPLICATIONS TO INTEGRAL GROUP RINGS OF DIHEDRAL AND QUATERNION GROUPS

BY

WILLIAM H. GUSTAFSON AND KLAUS W. ROGGENKAMP[1]

In Memoriam Irving Reiner

0. Introduction

In this paper, we show how Mayer-Vietoris sequences can be constructed to permit the computation of Picard groups and outer automorphism groups of orders from fibre product diagrams. We then illustrate the use of these sequences by carrying out the computations for certain group rings. The idea that there might be such sequences was inspired by the use of pullback methods in the construction [20] by the second author and L.L. Scott, Jr. of a counterexample to the Zassenhaus conjecture.

We feel that the mathematics in the paper is very much in the spirit of our dear friend and teacher, the late Irving Reiner. We humbly dedicate this work to his memory.

Let R be a Dedekind domain with field of fractions K; for instance, the ring of algebraic integers in the algebraic number field K. Let Λ be an R-order in a separable K-algebra A. For an R-subalgebra T of the center $Z(\Lambda)$ of Λ, we denote by $\mathrm{Pic}_T(\Lambda)$ the group of isomorphism classes $[M]$ of invertible Λ-bimodules with $tm = mt$ whenever $t \in T$ and $m \in M$. This group was first studied in the setting of orders by Fröhlich [3]. We shall consider the subgroup $\mathrm{LFPic}_T(\Lambda)$ of $\mathrm{Pic}_T(\Lambda)$ consisting of those $[M]$ for which M is locally free on one side. By a result of Swan [23], we have

$$\mathrm{Pic}_T(\mathbf{Z}G) = \mathrm{LFPic}_T(\mathbf{Z}G),$$

where $\mathbf{Z}G$ denotes the integral group ring of the finite group G.

Received July 20, 1987.

[1]During the preparation of this paper, this author was an ESA Visiting Scholar at Texas Tech University and was also supported by a grant from the Deutsche Forschungsgemeinschaft.