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A FINITENESS THEOREM FOR THE SPECTRAL SEQUENCE OF A RIEMANNIAN FOLIATION

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Introduction

Let M be a smooth closed manifold which carries a smooth foliation $\mathscr{F}$ of dimension p and codimension q. A differential form ω of degree r is said to be of filtration $\geq k$ if it vanishes whenever $r - k + 1$ of the vectors are tangent to $\mathscr{F}$. In this way the deRham complex of the differential forms becomes a filtered differential algebra and we have the spectral sequence $(E_i(\mathscr{F}), d_i)$ which converges after a finite number of steps to the (finite dimensional) cohomology of M.

It is clear that $E_2^{0,0}(\mathscr{F})$, $E_2^{1,0}(\mathscr{F})$, $E_2^{q-1,p}(\mathscr{F})$ and $E_2^{q,p}(\mathscr{F})$ are of finite dimension but there are another vectorial spaces $E_2^{u,v}(\mathscr{F})$ that may be infinite-dimensional as shown in the examples of G.W. Schwarz [7].

In [6], K.S. Sarkaria proves that $E_2(\mathscr{F})$ is finite-dimensional when $\mathscr{F}$ is transitive. He uses techniques of functional analysis (constructing a 2-parametrix).

In [2], A. El Kacimi-Alaoui, V. Sergiescu and G. Hector prove that the basic cohomology, [which is equal to $E_2^{\cdot,0}(\mathscr{F})$) is finite-dimensional. They prove it step to step for Lie foliations, transversely parallelizable foliations and Riemannian foliations.

This paper establishes the following improvement of the two results above.

THEOREM. *If a smooth closed manifold M carries a Riemannian foliation $\mathscr{F}$ then $E_2(\mathscr{F})$ is finite-dimensional.*

To prove it we assume that $\mathscr{F}$ is transversely oriented and construct an operation of a Lie algebra in $E_1(\hat{\mathscr{F}})$, where $\hat{\mathscr{F}}$ is the horizontal lift of $\mathscr{F}$ to the principal fiberbundle of oriented orthonormal frames with the transverse

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