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COMPLEMENTED COPIES OF c_0 IN l^∞-SUMS OF BANACH SPACES

BY

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1. Introduction

By a result of W.B. Johnson [6], there exists a sequence (G_n) of finite dimensional Banach spaces such that for any separable Banach space G, the l^∞-sum of (G_n) contains an isometric 1-complemented copy of G'. From this we see that the l^∞-sum of a family $(E_\gamma)_{\gamma\in\Gamma}$ of Banach spaces may contain many very different complemented infinite dimensional subspaces even if none of the spaces E_γ does. However, the situation becomes quite different when we consider complemented subspaces isomorphic to c_0. In fact, our main result shows that if the cardinality of the index set Γ is smaller than the first real-valued measurable cardinal, then the l^∞-sum of $(E_\gamma)_{\gamma\in\Gamma}$ contains a complemented copy of c_0 if and only if one of the spaces E_γ does. From this we obtain that the l^∞-sum of certain families of Grothendieck spaces is again a Grothendieck space.

Our terminology is standard. However, we want to explain some frequently used terms and fix some notations. For a (real) Banach space E we denote by E' the dual and by E'' the bidual of E. On E', the weak* and weak topologies are the topologies $\sigma(E', E)$ and $\sigma(E', E'')$ respectively. A sequence (x_i) in E is called *weakly summable* if

$$\sum |\langle x_i, x'\rangle| < \infty \quad \text{for all } x' \in E'.$$

E is said to contain a complemented copy of c_0 if E has a complemented subspace isomorphic to c_0. Given a set Γ, we denote by $|\Gamma|$ the cardinality of Γ. If $(E_\gamma)_{\gamma\in\Gamma}$ is a family of Banach spaces and $1 \le p \le \infty$, the l^p-sum of $(E_\gamma)_{\gamma\in\Gamma}$ is the space $(\Sigma \oplus E_\gamma)_{l^p(\Gamma)}$ which consists of all $x = (x(\gamma))_{\gamma\in\Gamma}$ such that $x(\gamma) \in E_\gamma$ for all γ and

$$\sum_{\gamma\in\Gamma} \|x(\gamma)\|^p < \infty \quad \left(\sup_\gamma \|x(\gamma)\| < \infty \text{ if } p = \infty\right),$$

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