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SOME BOUNDEDNESS RESULTS FOR ZERO-CYCLES ON SURFACES

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Introduction

Let X be a smooth, complex projective algebraic surface (which will be assumed throughout the rest of this paper). For any smooth variety V of dimension n, we will denote by $CH^k(V)$ the corresponding Chow group of algebraic cycles of codimension k in V (modulo *rational* equivalence), and write $CH_{n-k}(V) = CH^k(V)$. Our main focus of attention is on the subgroup $A_0(X)$ of zero-cycles of *degree* 0 in $CH_0(X)$, and more particularly on $T(X)$ = kernel of the *Albanese* map $\mathring{a}$: $A_0(X) \to Alb(X)$. Before stating the main theorem ((0.3)), we introduce the following terminology.

(0.1) DEFINITION. Let $B_0(X)$ be a subgroup of $A_0(X)$. We say that $A_0(X)/B_0(X)$ is finite dimensional if there exists a (possibly reducible) smooth curve E, a cycle z in $CH^2(E \times X)$ such that the composite

$$J(E) \xrightarrow{z_*} A_0(X) \longrightarrow A_0(X)/B_0(X)$$

is surjective.

Example. We can write $T(X) = A_0(X)/B_0(X)$ where $B_0(X)$ is defined as follows. By Poincaré's complete reducibility theorem, there exists an abelian variety B and a homomorphism f such that the composite

$$B \xrightarrow{f} A_0(X) \xrightarrow{\mathring{a}} Alb(X)$$

is an isogeny (see [8: (1.2)]). Clearly $T(X) + f(B) = A_0(X)$, moreover using $T(X)$ torsionless [7] it follows that $f(B) \cap T(X) = 0$. Now set $B_0(X) = f(B)$.

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