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LARGE CHARACTER DEGREES OF GROUPS OF ODD ORDER

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1. Introduction

In his paper [4], Gluck conjectures that the index of the Fitting subgroup of a finite solvable group G is bounded by the square of the largest degree of an irreducible character (over the complex field) of G, i.e.,

$$|G:F(G)| \leq b(G)^2 \quad \text{where } b(G) = \max\{\psi(1) | \psi \in \operatorname{Irr}(G)\}.$$

(See p. 447 of [4].)

He shows in Theorem B of [4] that $|G:F(G)| \leq b(G)^{13/2}$ holds for a solvable group G. Here we prove his conjecture with the additional assumption that $|G|$ is odd. As a corollary, we show that if G is a (solvable) group of odd order, then G has an abelian subgroup A such that $|G:A| \leq b(G)^6$. This improves (for groups of odd order) the bound $|G:A| \leq b(G)^{21/2}$ obtained in Theorem C of [4].

Our method consists of obtaining a regular orbit theorem for groups of odd order acting on vector spaces of odd characteristic (see Lemma 2.1 below). As Gluck points out, this method cannot be used for even order solvable groups (see p. 447 of [4]).

Our regular orbit theorem may be also used to prove a conjecture due to Huppert in some cases. Let G be a finite solvable group and, for a natural number n, let $\pi(n)$ denote the set of prime divisors of n. For a group H, we write $\pi(H)$ for $\pi(|H|)$. Define

$$\sigma(G) = \max_{\chi \in \operatorname{Irr}(G)} |\pi(\chi(1))|,$$

$$\rho(G) = \bigcup_{\chi \in \operatorname{Irr}(G)} \pi(\chi(1))$$

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