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QUATERNIONIC KAEHLER MANIFOLDS AND A CURVATURE CHARACTERIZATION OF TWO-POINT HOMOGENEOUS SPACES

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0. Introduction

This note is a continuation of the study in [9] on a conjecture of Bob Osserman:

Conjecture. A nonflat Riemannian manifold is locally symmetric of rank one if the curvature operator $K_v = R(\cdot, v)v$ for any unit vector v has constant eigenvalues, counting multiplicities.

The affirmation of this conjecture would give us a very geometric understanding of two-point homogeneous spaces, for which the curvature condition in the conjecture is automatically satisfied.

The author showed in [9] that the conjecture is true if the dimension of the manifold is 4, an odd number, or 2 times an odd number, or if it is a Kaehler manifold of nonnegative or nonpositive curvature. The Kaehler case is a direct consequence of a result of Bishop and Goldberg stating that the maximal (minimal resp.) sectional curvature at each point of a Kaehler manifold is holomorphic, provided the manifold is nonnegatively (nonpositively resp.) holomorphically pinched [6]. Upon noticing that the curvature condition in the above conjecture implies the manifold is locally irreducible (Lemma 2), it is natural to study the quaternionic case after the Kaehler one in view of the short list of Berger on the possible holonomy groups for irreducible spaces [2], [18].

A quaternionic Kaehler manifold of dimension $4n$, $n \geq 2$, is a Riemannian manifold whose holonomy group lies in $Sp(n) \cdot Sp(1) \subset SO(4n)$[1] [5], [10], [11], [12], [15]. Such spaces bear some resemblence to, though differ much from Kaehler manifolds due to the fact that the $Sp(1)$ part of the holonomy

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[1]Since $Sp(1) \cdot Sp(1) = SO(4)$, a quaternionic Kaehler manifold of dimension 4 is just a general Riemannian manifold. However the conjecture of Osserman is true in this case as mentioned earlier.