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A VERSALITY THEOREM FOR TRANSVERSELY HOLOMORPHIC FOLIATIONS OF FIXED DIFFERENTIABLE TYPE

BY

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Introduction

The aim of this paper is to give a versality theorem (analogous to that of Kuranishi for compact complex manifolds [6]) for transversely holomorphic foliations of fixed differentiable type (Theorem 7.1).

Throughout M will denote a compact C^∞-manifold endowed with a transversely holomorphic foliation $\mathscr{F}$ defined by a foliate cocycle $\{U_i, f_i, Z, \gamma_{ij}\}$, where $\{U_i\}$ is an open covering of M, f_i: $U_i \to Z$ are C^∞-submersions, Z is a complex manifold and $\{\gamma_{ij}\}$ are local holomorphic transformations of Z such that $f_i = \gamma_{ij} \circ f_j$. A family of deformations $\mathscr{F}^t$ of $\mathscr{F}$ parametrized by a germ of analytic space (T, o) is defined by a family of C^∞-submersions, f_i^t: $U_i \to Z$, parametrized by (T, o), depending holomorphically on t for each $x \in U_i$, and a family γ_{ij}^t of local holomorphic transformations of Z parametrized by the same (T, o), such that $f_i^t = \gamma_{ij}^t \circ f_j^t$, with $f_i^o = f_i$ and $\gamma_{ij}^o = \gamma_{ij}$. Two families of deformations, $\mathscr{F}^t$ and $\mathscr{F}'^t$, parametrized by the same (T, o) are said to be *isomorphic* if there exists a C^∞-family h^t of diffeomorphisms of M parametrized by (T, o), with $\mathscr{F}'^t = (h^t)^* \mathscr{F}^t$.

Girbau, Haefliger and Sundararaman [3] proved the existence of a germ of analytic space (S, o) (versal space) parametrizing a family of deformations $\mathscr{F}^s$ of $\mathscr{F}$ (versal family), with the following property: if $\mathscr{F}'^t$ is another family of deformations of $\mathscr{F}$ parametrized by (T, o), then there exists a morphism of germs of analytic spaces, f: $(T, o) \to (S, o)$, such that $\mathscr{F}^{f(t)}$ is isomorphic to $\mathscr{F}'^t$. Moreover the tangent map $d_o f$ of f at o is unique.

Most of the computable examples have a smooth versal space; that is, S is the germ at the origin of a complex vector space, concretely the cohomology space $H^1(M, \Theta^{tr})$, where Θ^{tr} is the sheaf of germs of local C^∞-vector fields generating flows preserving $\mathscr{F}$. There is a useful sufficiency criterion for the

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