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NOTE ON CONTINUITY OF INFORMATION RATE

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1. Introduction

The mutual information is a measure describing how much information carried by an input probability measure of an input system X is correctly transmitted to an output system Y through a channel λ. This quantity plays an essential role in communication theory since it is the measure that quantitively expresses efficiency of information transmission [9].

L. Breiman [3] showed that the information rate, i.e., the mutual information per unit time is upper semicontinuous in the vague topology of input probability measures for stationary channels with finite-memory. This result has been used [3] to show that the stationary capacity of such a channel coincides with the ergodic capacity of the channel (conjecture by Khinchin [9]). The continuity property of the information rate has been discussed as a basic property of information function. In this paper, we study this continuity with respect to several topologies in the space of input probability measures. In §2, we briefly review finite-alphabet, discrete-time communication processes and fix terminologies used here. In §3, we explain several topologies for the space of input probability measures and show the continuity of the information rate in the set-wise convergence of the input measures. In §4, we show that the information rate is not continuous in the cylinder-wise convergence of the input measures. In §5, for finite-memory stationary channels, we show the continuity with respect to Ornstein's $\bar{d}$-distance for the input measures. The $\bar{d}$-distance topology is weaker than the set-wise convergence and is stronger than the cylinder-wise convergence (see §6). In §6, we present an example of a non-finite-memory channel for which the information rate is discontinuous in the $\bar{d}$-distance. We give, also in §6, some remarks for the results obtained in §4 and §5.

2. Preliminaries

In information theory, a finite-alphabet and discrete-time communication process is mathematically formulated as follows [9]: Let A and B be finite

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