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PLURIHARMONIC SYMBOLS OF COMMUTING TOEPLITZ OPERATORS

BOO RIM CHOE AND YOUNG JOO LEE

1. Introduction and Results

Our setting throughout the paper is the unit ball B_n of the complex n-space $\mathbf{C}^n$; dimension n is fixed and thus we usually write $B = B_n$ unless otherwise specified. The Bergman space $A^2(B)$ is the closed subspace of $L^2(B) = L^2(B, V)$ consisting of holomorphic functions where V denotes the volume measure on B normalized to have total mass 1. For $u \in L^\infty(B)$, the *Toeplitz operator* T_u *with symbol* u is the bounded linear operator on $A^2(B)$ defined by $T_u(f) = P(uf)$ where P denotes the orthogonal projection of $L^2(B)$ onto $A^2(B)$. The projection P is the well-known Bergman projection which can be explicitly written as follows:

$$P(\psi)(z) = \int_B \frac{\psi(w)}{(1 - \langle z, w\rangle)^{n+1}}\, dV(w) \quad (z \in B)$$

for functions $\psi \in L^2(B)$. Here $\langle\, ,\rangle$ is the ordinary Hermitian inner product on $\mathbf{C}^n$. See [7, Chapters 3 and 7] for more information on the projection P.

In one dimensional case, Axler and Čučković [3] has recently obtained a complete description of harmonic symbols of commuting Toeplitz operators: if two Toeplitz operators with harmonic symbols commute, then either both symbols are holomorphic, or both symbols are antiholomorphic, or a nontrivial linear combination of symbols is constant (the converse is also true and trivial). Trying to generalize this characterization to the ball, one may naturally think of pluriharmonic symbols. A function $u \in C^2(B)$ is said to be *pluriharmonic* if its restriction to an arbitrary complex line that intersects the ball is harmonic as a function of single complex variable. As is well known, a real-valued function on B is pluriharmonic if and only if it is the real part of a holomorphic function on B. Hence every pluriharmonic function on B can be expressed, uniquely up to an additive constant, as the sum of a holomorphic function and an antiholomorphic function.

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