ILLINOIS JOURNAL OF MATHEMATICS
Volume 39, Number 2, Summer 1995

A GENERALIZED JENSEN'S INEQUALITY FOR BOUNDED ANALYTIC FUNCTIONS

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The classical Jensen's inequality, valid for any non-zero function in the Hardy space H^1 states that

$$\int_0^{2\pi} \log |f(e^{i\theta})| \frac{d\theta}{2\pi} \geq \log |f(0)|. \tag{1}$$

This estimate plays an essential rôle, for instance in number theory (see [5]), in conjunction with Mahler's measure:

$$M(f) = \exp \int_0^{2\pi} \log |f(e^{i\theta})| \frac{d\theta}{2\pi}.$$

Estimate (1) has several drawbacks. First, it takes into account only the 0-th coefficient of f. It is also discontinuous, in the sense that applying it to the sequence of functions $f_n(z) = 1/n + z$ leads to the estimate $\log 1/n \to -\infty$, though the limit function satisfies $\int \log |f| = 0$.

In order to find better versions of Jensen's inequality, one is naturally led to the concept of *concentration at low degrees*, introduced by Beauzamy-Enflo in [1]. Say that a polynomial $P(z) = \Sigma_0^n a_j z^j$ has concentration d $(0 < d \leq 1)$ at degree k if

$$\sum_0^k |a_j| \geq d \sum_0^n |a_j|. \tag{2}$$

It is possible to improve on Jensen's inequality by finding a constant $\tilde{C}(d, k)$ depending only on the concentration of the polynomial (and not on the degree) such that

$$\int_0^{2\pi} \log |P(e^{i\theta})| \frac{d\theta}{2\pi} \geq \tilde{C}(d, k). \tag{3}$$

Received November 30, 1992.
1991 Mathematics Subject Classification. Primary 30A10; Secondary 30C10.
[1]Supported by the C.N.R.S. (France) and the N.S.F. (U.S.A.), by contracts E.T.C.A./C.R.E.A. nos 20367/91 and 20388/92 (Ministry of Defense, France) and by research contract EERP-FR 22, Digital Equipment Corporation.