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NULL HOLOMORPHICALLY FLAT INDEFINITE ALMOST HERMITIAN MANIFOLDS

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1. Introduction

In this paper, we shall study the holomorphic sectional curvatures on indefinite almost Hermitian manifolds, with attention to the behaviour of the Jacobi operator along spacelike, timelike and null geodesics.

The study of sectional curvatures on manifolds with indefinite metrics exhibits significant differences from the positive definite case. In fact, at each point m of a Riemannian manifold M the sectional curvature is a function defined on the Grassmann manifold $G_2(T_m M)$ of planes on the tangent space $T_m M$ at m, and hence bounded. For a semi-Riemannian manifold M, however, $G_2(T_m M)$ at each point m contains degenerate planes, on which the sectional curvature is not defined. That is, the sectional curvature is well-defined only on the noncompact submanifold $G_2^0(T_m M)$ of $G_2(T_m M)$, which consists of all nondegenerate planes in $T_m M$. Thus the sectional curvature is not necessarily bounded.

It is a significant observation by Kulkarni [7] that boundedness of the sectional curvature on a semi-Riemannian manifold implies the constancy of the sectional curvature. It is elementary to recognize that if the sectional curvature is to be a function with definite values over $G_2(T_m M)$, then the curvature tensor R must satisfy the condition

$$R(x, y, x, y) = 0 \tag{1.1}$$

for any degenerate plane $\pi = \{x, y\} \in G_2(T_m M)$, where $x, y \in T_m M$ span the plane π. Dajczer and Nomizu [4] showed that such a condition implies the constancy of the sectional curvature on all nondegenerate planes. See also a work of Thorpe [14] for a Lorentz manifold.

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