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NORM STRUCTURE FUNCTIONS AND EXTREMENESS CRITERIA FOR OPERATORS ON $L_p(p \leq 1)$ OR ONTO $C(K)$

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Introduction

For a Banach or L_p space $\mathbf{E}$ and a Banach space $\mathbf{F}$, let $\mathscr{U} \equiv \mathscr{U}(\mathbf{E}, \mathbf{F})$ be the unit ball of the Banach space $\mathscr{L} \equiv \mathscr{L}(\mathbf{E}, \mathbf{F})$ of bounded linear operators T from $\mathbf{E}$ to $\mathbf{F}$. We study the extreme points of $\mathscr{U}$ when either (i) $\mathbf{E}$ is an L_p space ($p \leq 1$) or (ii) $\mathbf{F}$ is a C-space, i.e., a Banach space $C(K)$ of continuous functions on a compact Hausdorff space K. Extremeness criteria are obtained partly in terms of *norm structure functions* $\delta_1(T)$ and $\delta_\infty(T)$ for the cases (i) and (ii) respectively. The first function $\delta_1(T)$ generalizes the function $|T|^*1$ for the case where $\mathbf{E}$ and $\mathbf{F}$ are L_1 spaces, and the second, $\delta_\infty(T)$, generalizes $|T|1$ for the case where both spaces are C-spaces. Some of their basic properties are studied that are used in tackling the extremeness problems. The scalar field may be the reals or the complexes. The proofs are given for the complex case; the real case follows by minor adjustments.

In the case (i) we obtain, among other things, complete description of extreme contractions in $\mathscr{U}(L_p(\mu), L_q(\nu))$ when $0 < p \leq 1 \leq q < \infty$ in a rather unified manner (Theorem 2.8). Some of our extremeness results for the case $\mathbf{E} = L_1(\mu)$ have points of contact with some results of [Sh2, §2] but the approach and formulation are different. When the scalars are the reals, special cases for $\mathbf{E} = L_1(\mu)$ and $\mathbf{F} = L_1(\nu)$ have been considered in [I, Theorem 2] and, implicitly, in [Ki, Theorem 2]. Concerning case (ii) the problem of characterizing an extreme contraction T between C-spaces $\mathbf{E} = C(H)$ and $\mathbf{F} = C(K)$ have been studied by several researchers. The most desirable criterion for T to be extreme seems to be that T be a composition operator modulated by a unimodular function, which is just criterion $(\infty\infty)'$ in Theorem 3.7. This is equivalent, as is not difficult to show, to T^* mapping all extreme points of the unit ball of $\mathbf{F}'$ to those of that of $\mathbf{E}'$; such a T is said to be *nice*. (The extreme points mentioned are unimodular scalar multiples of evaluation maps.) The criterion, clearly sufficient, is not always necessary

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