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BOUNDEDNESS OF SOME SUBLINEAR OPERATORS ON HERZ SPACES

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1. Introduction

It is well known that Beurling [2] and Herz [11] introduced some new spaces that characterize certain properties of functions. These new spaces are called the Herz spaces. Many studies involving these spaces can be found in the literature. One of the main reasons is that Hardy space theory associated with Herz spaces is very rich. Actually, these new Hardy spaces are a sort of local version of the ordinary Hardy spaces; the former, sometimes, are good substitutes of the latter when considering, for example, the boundedness of non-translation invariant singular integral operators. This paper is motivated by previous work of Lu, Hernández and the second author (see [14] and [10]), and also by more applications, such as the boundedness of bilinear operators and the regularity of solutions of the Laplacian and the wave equations on Herz-type spaces. See [12] and [16]. Our main interest is to study the boundedness of some sublinear operators on these spaces under certain weak size conditions (see (2.1) and (2.2) below). These conditions are similar to those introduced by Soria and Weiss in [18], and are satisfied by most of the operators in harmonic analysis (see [18]). Let us first introduce some notations.

Let $B_k = \{x \in \mathbb{R}^n\colon |x| \leq 2^k\}$ and $A_k = B_k \setminus B_{k-1}$ for $k \in \mathbb{Z}$. Let $\chi_k = \chi_{A_k}$ for $k \in \mathbb{Z}$, where χ_E is the characteristic function of the set E.

Definition 1.1. Let $\alpha \in \mathbb{R}$, $0 < p \leq \infty$ and $0 < q \leq \infty$.

(a) The homogeneous Herz space $\dot{K}_q^{\alpha,p}(\mathbb{R}^n)$ is defined by

$$\dot{K}_q^{\alpha,p}(\mathbb{R}^n) = \{f \in L_{\mathrm{loc}}^q(\mathbb{R}^n \setminus \{0\})\colon \|f\|_{\dot{K}_q^{\alpha,p}(\mathbb{R}^n)} < \infty\},$$

where

$$\|f\|_{\dot{K}_q^{\alpha,p}(\mathbb{R}^n)} = \left\{ \sum_{k=-\infty}^{\infty} 2^{k\alpha p} \|f\chi_k\|_{L^q(\mathbb{R}^n)}^p \right\}^{1/p} < \infty,$$

with the usual modifications made when $p = \infty$ and/or $q = \infty$.

(b) The non-homogeneous Herz space $K_q^{\alpha,p}(\mathbb{R}^n)$ is defined by

$$K_q^{\alpha,p}(\mathbb{R}^n) = \{f \in L_{\mathrm{loc}}^q(\mathbb{R}^n)\colon \|f\|_{K_q^{\alpha,p}(\mathbb{R}^n)} < \infty\},$$

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