ILLINOIS JOURNAL OF MATHEMATICS
Volume 41, Number 3, Fall 1997

SIMPLICIAL CURRENTS[1]

JOHAN L. DUPONT AND HENRIK JUST

0. Introduction

For a smooth manifold X, the deRham theorem provides a quasi-isomorphism from the complex $\Omega^*(X)$ of differential forms to the complex of (smooth) singular cochains on X. Furthermore (under this isomorphism) the wedge-product in $\Omega^*(X)$ induces the cup-product in cohomology; but $\Omega^*(X)$ has the advantage of being an associative, graded commutative algebra already on the chain level.

In the dual case the deRham theorem gives a quasi-isomorphism from the complex of (smooth) singular chains on X to the complex $\Omega_*(X)$ of compactly supported currents on X. (We use this non-standard notation rather than $\mathcal{D}'(X)$ or $\mathcal{D}'_*(X)$.) The dual of the wedge-product is a map

$$\wedge' \colon \Omega_*(X) \to \Omega_*(X) \hat{\otimes} \Omega_*(X)$$

(where $\hat{\otimes}$ denotes the completed, projective tensor-product), and this is in the appropriate sense an associative and graded commutative coproduct. Furthermore there is a commutative diagram

$$\begin{tikzcd} H(\Omega_*(X)) \arrow[r, "\wedge'"] \arrow[d, equal] & H(\Omega_*(X)\hat{\otimes}\Omega_*(X)) \\ H(\Omega_*(X)) \arrow[r, "\text{coproduct}"] & H(\Omega_*(X)) \otimes H(\Omega_*(X)) \arrow[u, "\cong"'] \end{tikzcd} \tag{0.1}$$

proving that $\wedge'$ identifies with the usual coproduct in homology.

The deRham theorem has a natural and frequently used extension to the category of simplicial manifolds, i.e., simplicial objects in the differentiable category. Here the complex $\Omega^* \|X\|$ of simplicial differential forms, as defined in [5], plays the role of the differential forms on a manifold. That is, $\Omega^*\|X\|$ is an associative, graded commutative algebra, and the cohomology identifies with the cohomology algebra of the (fat) realization $\|X\|$ (see Section 3 for the definitions).

The aim of the following is to introduce a complex $\Omega_* \|X\|$ of *simplicial currents* on a simplicial manifold X, with properties similar to the complex of currents on

Received April 3, 1995.
1991 Mathematics Subject Classification. Primary 58A12, 58A25; Secondary 14F40, 55P62, 57P99.
[1]Work supported in part by Statens Naturvidenskabelige Forskningsråd, Denmark and the Rosenbaum Foundation, U.S.A.