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LINEAR ISOMETRIES BETWEEN CERTAIN SUBSPACES OF CONTINUOUS VECTOR-VALUED FUNCTIONS

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Introduction

Throughout this note, X and Y will stand for locally compact Hausdorff spaces, and E and F for Banach spaces. Let $C_0(X, E)$ and $C_0(Y, F)$ be the Banach spaces of continuous E-valued and F-valued functions vanishing at infinity defined on X and Y respectively and endowed with the supremum norm $\|\cdot\|_\infty$. Let $\mathbf{K}$ denote the field of real or complex numbers. If $E = F = \mathbf{K}$, then we will write $C_0(X)$ and $C_0(Y)$ ($C(X)$ and $C(Y)$ if X, Y are compact).

The classical Banach-Stone theorem states that if there exists a linear isometry T of $C(X)$ onto $C(Y)$, then there is a homeomorphism ψ of Y onto X and a continuous map a: $Y \to \mathbf{K}$, $|a| \equiv 1$, such that T can be written as a weighted composition map; that is,

$$(Tf)(y) = a(y)f(\psi(y)) \text{ for all } y \in Y \text{ and all } f \in C(X).$$

An important generalization of the Banach-Stone theorem was given by W. Holsztyński in [9] by considering non-surjective isometries. Namely, he proved that, in this case, there is a closed subset Y_0 of Y where the isometry can still be represented as a weighted composition map. Recently, in [1], the authors have widely generalized this result by studying linear isometries between certain subspaces of $C_0(X)$ and $C_0(Y)$.

In the context of continuous vector-valued functions similar results are available. In [10], M. Jerison investigated the vector analogue of the Banach-Stone theorem: If X and Y are compact Hausdorff spaces and E is a strictly convex Banach space, then every linear isometry T of $C(X, E)$ onto $C(Y, E)$ can be written as a weighted composition map; namely, $(Tf)(y) = \omega(y)(f(\psi(y)))$, for all $f \in C(X, E)$ and all $y \in Y$, where ω is a continuous map from Y into the space of linear isometries from E onto E endowed with the strong operator topology. Furthermore, ψ is a homeomorphism of Y onto X. As in the scalar-valued case, Jerison's results have been extended in many directions (e.g., see [3] or [4]). Among them and in [6], M. Cambern obtained a formulation of Holsztyński's theorem for spaces of continuous vector-valued functions.

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