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ON THE SEQUENCES THAT ARE GOOD IN THE MEAN FOR POSITIVE L_p-CONTRACTIONS, $1 \leq p < \infty$

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1. Introduction

It is a well-known fact that, if a weight **n** is good a.e. for all operators induced by measure preserving transformations (MPTs), then it is also good a.e. for any Dunford-Schwartz operator (i.e., $L_1 - L_\infty$-contraction) [BO]. Similar results have been obtained in various other settings [JO], [JOW], [ÇLO]. When T is an L_1-contraction induced by an MPT, various types of sequences, such as $\mathbb{Z}$, block sequences [BL], sequences satisfying the cone condition [BL], [RW], sequence of squares and sequence of primes [RW] are good in the mean for T. Recently, it was proved in [F] that, sequences satisfying the cone condition are good a.e. and in the mean for the class of bounded superadditive processes relative to MPTs.

In this article, our aim is to show that sequences which are good in the mean for invertible MPTs are also good in the mean for T-(super)additive processes relative to positive L_p-contractions (when $1 < p < \infty$), or positive Dunford-Schwartz operators on L_1.

Let (X, Σ, μ) be a finite measure space, and let $T\colon L_p(X) \longrightarrow L_p(X)$ be a positive linear contraction where $1 \leq p < \infty$ is fixed. In order to avoid certain difficulties we will assume that (X, Σ, μ) is a Lebesgue space. A strictly increasing sequence $\mathbf{n} = \{n_k\}$ of integers is called *good in the p-mean for T* if, for every $f \in L_p$, $\lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N-1} T^{n_i} f$ exists in the L_p-norm . If τ is a measurable transformation on X, we say that **n** is *good in the p-mean for τ* when it is good in the p-mean for the operator T induced by τ. As usual, **n** is called *good in the p-mean* if it is good in the p-mean for all MPTs.

A family $F = \{F_n\}_{n\geq 0}$ of functions in L_p is called a *T-superadditive process* if $F_{n+m} \geq F_n + T^n F_m$ a.e. for all $n, m \geq 0$ ($F_0 = 0$), where T is a positive linear operator on L_p. If the reverse inequality holds, it is called *T-subadditive*, and if the equality holds, i.e., $F_n = \sum_{i=0}^{n-1} T^i F_1$, it is called *T-additive*. A nonnegative T-superadditive process F is called *bounded* if $\gamma_F = \sup_{n\geq 1} \frac{1}{n} \int F_n d\mu < \infty$. It is well known that, if $F \subset L_1^+$ is bounded, then $\lim_{n\to\infty} \frac{1}{n} \int F_n d\mu = \gamma_F$.

In order to define the "averages" of a T-superadditive process $F = \{F_n\}$ along a general sequence **n**, it will be convenient to view F as a collection of functions $\{f_k\}$

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