LIKELIHOOD RATIOS FOR STOCHASTIC PROCESSES RELATED BY GROUPS OF TRANSFORMATIONS

BY

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1. Introduction

If $x(t)$ and $y(t)$ are stochastic processes with the same parameter set, they induce measures m_x and m_y on a suitably chosen space of sample functions. It is an important problem of statistics to find conditions guaranteeing the existence of the Radon-Nikodym derivative (or likelihood ratio) dm_x/dm_y and to find formulas for computing it. These derivatives are also helpful in describing one process in terms of the other, in particular, in carrying almost everywhere properties from one process to another which is less well known.

This problem has been studied most in the case where $x(t)$ and $y(t)$ are closely related to a Brownian-motion process (see, for example, [1], [2], [7], [10], and [11]). Prokhorov [9] and Skorokhod [12] have investigated the case where $x(t)$ and $y(t)$ are solutions of a diffusion equation (again, of course, closely related to Brownian motion), and Skorokhod [13] has also investigated the case where $x(t)$ and $y(t)$ are processes with independent increments. The most important case in engineering applications is that for which the processes are Gaussian. This has been attacked by, among others, Grenander [6], Slepian [14], Feldman [5], and Woodward [15].

In most of the above work the special nature of the processes involved is relied on, in particular, the independence or near independence of many of the random variables arising in the computations. In this paper we shall develop a technique relying less on such computations and more on assumed geometrical relationships between the processes. This technique has already been applied in [8] to the mean value problem, $y(t) = x(t) + f(t)$ for a fixed $f(t)$ when $x(t)$ is the solution of a diffusion equation.

Throughout Sections 2 and 3 we shall make the following assumptions. We assume given a set X, a σ-algebra S of subsets of X, a probability measure P on (X, S), an algebra F of bounded, real-valued S-measurable functions containing the constant functions, and a one-parameter group T_α of automorphisms of F. F and T_α are to satisfy

(1) T_α preserves bounds and $T_\alpha f(x)$ has a continuous derivative which is bounded uniformly in α and x for every f in F and x in X.

(2) If f_n is a uniformly bounded sequence from F with $\lim f_n(x) = 0$ for all x, then $\lim T_\alpha f_n(x) = 0$ for all x.

(3) There exists a function ϕ in some $L_p(P)$, $1 \leqq p < \infty$, satisfying, for

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