# EXTENSIONS AND COROLLARIES OF RECENT WORK ON HILBERT'S TENTH PROBLEM[1]

BY

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This paper consists of three separate notes related only in that each of the three either extends or employs the results of [2], with which acquaintance is assumed.

## 1. A sharpening of Kleene's normal form theorem

By a form of Kleene's normal form theorem (cf. [1] or [3]) we may understand the following assertion:

THEOREM. *There is a function $U(y)$ and a predicate $T(z, x, y)$ both belonging to the class $Q$ such that a function $f(x)$ is partially computable if and only if for some number $e$*

$$f(x) = U(\min_y T(e, x, y)).$$

In its original form, this result was stated with $Q$ the class of primitive recursive functions and predicates. It is well known (cf. [3] and [6]) that smaller classes $Q$ suffice. We wish to point out here that (assuming variables to range over the positive integers) we may take for $Q$ the following extremely modest class:

(1) *A function $f$ belongs to $Q$ if and only if $f$ can be obtained by repeated application of the operation of composition to the functions: $2^x$, $x \cdot y$, $N(x) = 0$, $U_i^n(x_1, \cdots, x_n) = x_i$, $K(x)$, $L(x)$, where $K(x)$, $L(x)$ are recursive pairing functions.*

(2) *A predicate $R(x_1, \cdots, x_n)$ belongs to $Q$ if*

$$R(x_1, \cdots, x_n) \leftrightarrow f(x_1, \cdots, x_n) = g(x_1, \cdots, x_n)$$

*where $f, g \in Q$.*

In fact, we may even take $U(y) = K(y)$.

To see this we begin by noting that by Corollary 5 of [2], (or rather the immediate extension thereof to predicates), we have

$$\begin{aligned} \bigvee_y T_2(z, x, u, y) &\leftrightarrow \bigvee_{x_1, \cdots, x_n} P(z, x, u, x_1, \cdots, x_n, 2^{x_1}, \cdots, 2^{x_n}) = 0 \\ &\leftrightarrow \bigvee_{x_1, \cdots, x_n} \{ \textstyle\sum_{j=1}^m f_j(z, x, u, x_1, \cdots, x_n, 2^{x_1}, \cdots, 2^{x_n}) \\ &\qquad = \textstyle\sum_{j=1}^m g_j(z, x, u, x_1, \cdots, x_n, 2^{x_1}, \cdots, 2^{x_n}) \}, \end{aligned}$$

Received November 22, 1961.

[1] This research was supported by the United States Air Force through the Air Force Office of Scientific Research of the Air Research and Development Command.