SYMMETRIC HOMOLOGY SPHERES

BY
BARRY MAZUR

1. Introduction

In the course of thinking about a very suggestive conjecture [1], [2] concerning periodic transformations on the three-sphere, I ran across some interesting four-manifolds, W, which are of the homotopy type of S^4, but possibly not topologically equivalent to S^4. The conjecture claims that if a periodic transformation on the three-sphere has a circle as fixed-point set, then that circle must be unknotted. On these four-manifolds, W, that are constructed, one may exhibit an action of the circle group S, with fixed-point set a two-sphere Σ. The fundamental group of the complement, $\pi_1(W - \Sigma)$ is a split extension of the integers by a nontrivial group π, and therefore the two-sphere is knotted. (It cannot bound a flat disc.) The two-sphere Σ does however bound a one-parameter family of Poincaré cells (i.e., manifolds with trivial homology and with π as fundamental group) whose interiors are disjoint and which sweep out the space W.

The construction of these manifolds W involves the use of homology spheres with specific kinds of symmetries. Manifolds of that sort, I call symmetric homology spheres. The Poincaré icosahedral space is an example of such an object.

By employing a recent (as yet unpublished) characterization of Euclidean n-space ($n \geqq 5$) by Stallings, and using the above construction, an action of the circle on S^5 may be obtained, with a knotted three-sphere Σ^3 as fixed-point set, whose knot group is again a split extension of the group of integers by the group, π.

It should also be remarked that π may be taken to be the icosahedral group, thus exhibiting a phenomenon which cannot occur with knotted imbeddings of S^1 in S^3: the knot group of Σ^3 contains elements of finite order.

2. Terminology

All manifolds and maps in this paper will be combinatorial. Thus homeomorphism will mean combinatorial homeomorphism.

I denotes the unit interval, D^n the n-cell, S^n the n-sphere. If M is an n-manifold, ∂M is its boundary and int M its interior. M^* will denote M with a point removed; M_0 will denote M with the interior of a closed n-cell removed. A flat disc D^k in M^n is one which may be thrown onto the standard k-cell in a closed n-cell $D^n \subset M^n$ by a global automorphism of M^n. If X, Y are spaces, $f: X \rightarrow Y$ a map, $f: \pi_1(X) \rightarrow \pi_1(Y)$ will be the induced homo-

Received February 16, 1961.