SOLUTION OF CERTAIN NONAUTONOMOUS DIFFERENTIAL SYSTEMS BY SERIES OF EXPONENTIAL FUNCTIONS

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1. Introduction

In a recent paper [1] Wasow investigated systems of differential equations of the form

$$y' = f(y) + \sum g_k e^{i\omega_k x}. \tag{1.1}$$

Here y is an n-dimensional vector; y' denotes the derivative of y with respect to x; the g_k are constant vectors; the ω_k are real, not necessarily rationally independent numbers; the components of the vector $f(y)$ are assumed to be analytic functions of the components of y vanishing for $y = a$ and holomorphic in the neighborhood of a. The sum in (1.1) has $m < \infty$ terms.

Wasow constructs a solution of (1.1) of the form

$$y = a + \sum a_r e^{i\mu_r x}, \tag{1.2}$$

where the series converges uniformly and absolutely for $-\infty < x < \infty$ provided the coefficients g_k of (1.1) are sufficiently small. The numbers μ_r are linear combinations of the $\omega_1, \omega_2, \cdots, \omega_m$ with nonnegative integral coefficients, and the a_r are determined recursively by solving n^{th} order linear systems of equations. The individual terms of series (1.2) represent the theoretically and experimentally well-known combination harmonics in the response of system (1.1).

In this paper Wasow's results will be extended in several directions. The exponential polynomial of (1.1) will be replaced by a general exponential series

$$\sum_{k=1}^{\infty} g_k e^{i\omega_k x} \tag{1.3}$$

which includes the general almost periodic function and the general periodic function with absolutely convergent Fourier series. It will be shown that (1.2) is a real solution provided (1.1) is a real system. The general system

$$y' = g(x, y) \tag{1.4}$$

will be shown to have a solution of form (1.2) if the components of $g(x, y)$ are analytic functions of y, holomorphic for $\| y - a \| \leqq \rho$, with coefficients that are of the form (1.3).

In the next section it will be proved that if the linear homogeneous part of (1.1) has a solution of the form

$$a_0 e^{i(\mu_0/q)x}, \tag{1.5}$$

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