A FINE-CYCLIC ADDITIVITY THEOREM FOR A FUNCTIONAL[1]

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Introduction

Let J be a closed finitely connected Jordan region, and let (T, J) be a continuous mapping from J into E_3. L. Cesari has introduced in his papers [2; 3] the concept of a fine-cyclic element of (T, J), and he has proven that the Lebesgue area is fine-cyclicly additive, thus extending a well-known cyclic additivity theorem for the Lebesgue area [8]. A fine-cyclic element is actually a decomposition of a proper cyclic element, and, in case J is a 2-cell, is equivalent to a proper cyclic element.

In [6] a B-set and a fine-cyclic element of a Peano space is introduced as a generalization of an A-set and a proper cyclic element. Specifically, a B-set of a Peano space P is a nondegenerate (more than one point) continuum of P such that either $B = P$ or else every component of $P - B$ has a finite frontier. A fine-cyclic element of P is a B-set of P whose connection is not destroyed by removing any finite set. It has been shown in [6] that in a Peano space P whose degree of multicoherence $r(P)$ is finite, B-sets and fine-cyclic elements possess essentially the same properties as A-sets and proper cyclic elements.

In this paper a generalization of Cesari's fine-cyclic additivity theorem for the Lebesgue area is studied. The generalization proceeds along lines similar to [4] by considering nonnegative functionals Φ defined for each continuous mapping T from a Peano space P into a metric space P^*. Let $T = sf$, $f:P \to M$, $s:M \to P^*$, $r(M) < \infty$, be an unrestricted factorization of T (§1), and let $\{\Delta\}$ be the collection of fine-cyclic elements of M. With each Δ there is associated a connected open set $G_\Delta \subset M$ containing Δ such that Δ is a (G_Δ, A)-set [7]. Denote by t_Δ the natural retraction [7] from G_Δ onto Δ, and let $A_\Delta = f^{-1}(G_\Delta)$. If Φ satisfies the conditions of §2, the main result of this paper states that $\Phi(T, P) = \sum \Phi(st_\Delta f, A_\Delta)$, $\Delta \subset M$.

1. Mappings

Let P be a Peano space, and let P^* be a metric space. Denote by $\mathfrak{A}$ the collection of all open subsets of P. Let $\mathfrak{T}^*$ be the class of all continuous mappings (T, A) from any $A \in \mathfrak{A}$ into P^*. The subclass of $\mathfrak{T}^*$ consisting of all mappings (T, P) from P into P^* will be designated by $\mathfrak{T}$. It is well-known that each $(T, P) \in \mathfrak{T}$ admits of a monotone-light factorization [10]. However, this paper is independent of this particular factorization of (T, P), and hence we will consider *unrestricted* factorizations [4].

Received June 20, 1957.

[1] The present research was partially supported by ARDC.