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ON THE FUTAKI INVARIANTS OF COMPLETE INTERSECTIONS

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1. Introduction. In 1983, Futaki [2] introduced his invariants that generalize the obstruction of Kazdan-Warner to prescribe Gauss curvature on S^2. The Futaki invariants are defined for any compact Kähler manifold with positive first Chern class that has nontrivial holomorphic vector fields. Their vanishing is a necessary condition to the existence of Kähler-Einstein metrics on the underlying manifold.

Let M be a compact Kähler manifold with positive first Chern class $c_1(M) > 0$. Choosing an arbitrary positive $(1,1)$-form ω in $c_1(M)$ as a Kähler metric on M, we can find a smooth function f on M, determined up to a constant, such that

$$\operatorname{Ric}(\omega) - \omega = \frac{\sqrt{-1}}{2\pi}\partial\bar{\partial} f \tag{1.1}$$

holds. Let $\mathfrak{h}(M)$ be the Lie algebra of holomorphic vector fields on M. The Futaki invariants are defined as

$$F : \mathfrak{h}(M) \to C, \qquad F(X) = \int_M X(f)\omega^n.$$

Ding and Tian [1] introduced the Futaki invariants for Fano normal varieties. This is a generalization of Futaki invariants to singular varieties. It also has important application in Kähler-Einstein geometry. The Futaki invariants on singular varieties are related to the stability of Fano manifolds due to the work of Tian [8]. To be more precise, checking the K-stability of a Fano manifold is the same as checking the sign of the real part of the Futaki invariants on the degenerations of the Fano manifold. Because of this, we need an effective way to compute the Futaki invariants on singular varieties.

In this paper, we give a simple formula for the Futaki invariants of Fano complete intersections. The main theorem of this paper is the following.

THEOREM 1.1. *Let M be the $N-s$ dimensional normal Fano variety in CP^N defined by the homogeneous polynomials $F_1, \ldots, F_s$ of degree $d_1, \ldots, d_s$, respectively. Let X be a holomorphic vector field on CP^N such that*

$$XF_i = \kappa_i F_i, \quad i = 1, \ldots, s, \tag{1.2}$$

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