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LIFTING CUSP FORMS ON GL_{2n} TO $\widetilde{\mathrm{Sp}}_{2n}$: THE UNRAMIFIED CORRESPONDENCE

DAVID GINZBURG, STEPHEN RALLIS, AND DAVID SOUDRY

0. Introduction. Let τ be an irreducible, automorphic, cuspidal, self-dual representation of $\mathrm{GL}_{2n}(\mathbb{A})$, where $\mathbb{A}$ is the adele ring of a number field F. Assume that the partial exterior square L-function $L^S(\tau, \Lambda^2, s)$ has a pole at $s = 1$ and that the standard L-function $L(\tau, (1/2)) \neq 0$. In [GRS1] we constructed a space $\pi_\psi(\tau)$ of cusp forms on the metaplectic cover $\widetilde{\mathrm{Sp}}_{2n}(\mathbb{A})$. This space is invariant to right translations by $\widetilde{\mathrm{Sp}}_{2n}(\mathbb{A})$. It depends on a choice of a nontrivial character ψ of $F\backslash\mathbb{A}$. We proved in [GRS2, Chapter 5] that $\pi_\psi(\tau)$ is nontrivial. $\pi_\psi(\tau)$ affords a cuspidal representation of $\widetilde{\mathrm{Sp}}_{2n}(\mathbb{A})$, which we continue to denote by $\pi_\psi(\tau)$. We do not know yet how to prove that $\pi_\psi(\tau)$ is irreducible. Each irreducible summand π of $\pi_\psi(\tau)$ has a nontrivial ψ-Whittaker coefficient in the sense that

$$\int_{V_n(F)\backslash V_n(\mathbb{A})} \varphi(vg)\psi_n(v)\,dv \not\equiv 0 \tag{0.1}$$

as φ varies in (the space of) π. Here V_n denotes the standard maximal unipotent subgroup of Sp_{2n}, and ψ_n is the standard nondegenerate character of $V_n(\mathbb{A})$ which corresponds to ψ. By [GRS1, Section 2.3, Remark 1], $\pi_\psi(\tau)$ contains the direct sum of all ψ^{-1}-generic (i.e., satisfying (0.1)) cuspidal representations π of $\widetilde{\mathrm{Sp}}_{2n}(\mathbb{A})$, such that $L_\psi^S(\pi \otimes \tau, s)$ has a pole at $s = 1$. See [GRS3] for the definition of the standard L-function for generic representations of $\widetilde{\mathrm{Sp}}_{2n} \times \mathrm{GL}_k$ associated to ψ. The existence of the above pole indicates that τ is the "ψ-functorial lift of π," in the sense that for almost all places ν, where π_ν and τ_ν are unramified, we have

$$L_{\psi_\nu}(\pi_\nu, s) = L(\tau_\nu, s). \tag{0.2}$$

This is the main result of this paper. More precisely, here is the main theorem.

MAIN THEOREM. *The representation $\pi_\psi(\tau)$ is the direct sum of all irreducible, cuspidal, ψ^{-1}-generic representations π of $\widetilde{\mathrm{Sp}}_{2n}(\mathbb{A})$ that satisfy (0.2) at almost all places.*

We conjecture that $\pi_\psi(\tau)$ is irreducible. See [GRS1, Section 2.3]. Recall that the cusp forms of $\pi_\psi(\tau)$ are linear combinations of certain Fourier-Jacobi coefficients of

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