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CONSTANT SCALAR CURVATURE METRICS WITH ISOLATED SINGULARITIES

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1. Introduction and statement of the results. In this paper, we construct solutions of the Yamabe problem on the sphere (S^N, g_0) with its standard metric, that are singular at a specified closed set $\Lambda \subset S^N$. More specifically, we seek a new metric g that is conformal to g_0, is complete on $\Lambda \subset S^N$, and has constant positive scalar curvature R. The problem may be translated into a differential equation as follows. Since g is conformal to g_0, we may write $g = u^{4/(N-2)} g_0$ where u is a positive function on $M \setminus \Lambda$. The scalar curvature functions $R(g_0)$ of g_0 and $R(g)$ of g are related by the equation

$$\Delta_{g_0} u - \frac{N-2}{4(N-1)} R(g_0) u + \frac{N-2}{4(N-1)} R(g) u^{(N+2)/(N-2)} = 0. \tag{1}$$

In order that g be complete on $S^N \setminus \Lambda$, it is necessary for u to tend to infinity rather strongly on approach to Λ, and of course we wish to solve this equation with R a (prescribed) constant.

The first two terms of the operator on the left in (1), namely,

$$\mathscr{L}_{g_0} \equiv \Delta_{g_0} - \frac{N-2}{4(N-1)} R(g_0), \tag{2}$$

give a second-order, linear, elliptic differential operator, known as the conformal Laplacian of the metric g_0. It satisfies the conformal equivariance property that if two metrics are conformally related, such as g and g_0, then for any function ϕ,

$$\mathscr{L}_{g_0}(u\phi) = u^{(N+2)/(N-2)} \mathscr{L}_g(\phi). \tag{3}$$

Notice that (1) corresponds to the special case of (3) when $\phi = 1$.

This "singular Yamabe problem" has been extensively studied in recent years, as has the case when the ambient manifold is more general than the sphere; many existence results, as well as obstructions to existence, are known. Briefly, for a solution to exist on a general compact Riemannian manifold (M, g_0), the size of Λ and the sign of R must be related to one another. If a solution exists with $R < 0$, then $\dim(\Lambda) > (N-2)/2$, while if a solution exists with $R \geq 0$, then $\dim(\Lambda) \leq (N-2)/2$, and in

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