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PSEUDOHOLOMORPHIC CURVES AND THE SHADOWING LEMMA

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1. Introduction. Let M be a compact smooth manifold of dimension n, and let $T^*M \to^{\tau} M$ be its cotangent bundle. T^*M carries a canonical 1-form θ, which in canonical coordinates (q_i, p_i) is given by

$$\theta = \sum p_i \, dq_i.$$

Then $\omega := d\theta$ is a symplectic form on T^*M.

To a smooth Hamiltonian $H \in C^\infty(S^1 \times T^*M, \mathbb{R})$, 1-periodic in time, we associate the Hamiltonian system

$$\dot{x} = X_H(t, x), \tag{HS}$$

where the Hamiltonian vector field X_H is defined by

$$d_x H(t, x) = \omega\big(X_H(t, x), \cdot\big).$$

We make the following assumptions on H.

(H1) (Saddle point). There exists a point $x_0 = (q_0, 0) \in T^*M$ such that $H(t, x_0) = 0$, $d_x H(t, x_0) = 0$ for all t, and

$$\frac{\partial^2 H}{\partial p^2}(t, x_0)_{pp} > 0 \quad \text{for all } t\ , \qquad H(t, q_0, p) \geq 0 \quad \text{for all } t, p,$$
$$\frac{\partial^2 H}{\partial q^2}(t, x_0)_{qq} < 0 \quad \text{for all } t, \qquad H(t, q, 0) < 0 \quad \text{for all } q \neq q_0.$$

(H2) (Growth conditions). We have that

(i) $|d_x H(t, x)| \leq a\, d(x, x_0)$;

(ii) $H(t, q, p) \geq b_1 |p|^2 - b_2$;

(iii) there exists a vector field η on T^*M satisfying

$$d(i_\eta \omega) = \omega,$$
$$|\eta(x)| \leq c_1\, d(x, x_0),$$
$$d_x H(t, x) \cdot \eta(x) - H(t, x) \geq c_2 \big(d(x, x_0)\big)^2.$$

Here, a, b_i, c_i are positive constants; we have chosen a Riemannian metric on M and

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