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DISCRIMINANT COMPLEMENTS AND KERNELS OF MONODROMY REPRESENTATIONS

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1. Introduction. A hypersurface of degree d in a complex projective space $\mathbb{P}^{n+1}$ is defined by an equation of the form

$$F(x) = \sum a_L x^L = 0, \tag{1.1}$$

where $x^L = x_0^{L_0} \cdots x_{n+1}^{L_{n+1}}$ is a monomial of degree d and where the a_L are arbitrary complex numbers, not all zero. Viewed as an equation in both the a's and the x's, (1.1) defines a hypersurface $\mathbf{X}$ in $\mathbb{P}^N \times \mathbb{P}^{n+1}$, where $N+1$ is the dimension of the space of homogeneous polynomials of degree d in $n+2$ variables, and where the projection p onto the first factor makes $\mathbf{X}$ into a family with fibers $X_a = p^{-1}(a)$. This is the universal family of hypersurfaces of degree d and dimension n. Let Δ be the set of points a in $\mathbb{P}^N$ such that the corresponding fiber is singular. This is the *discriminant locus*; it is well known to be irreducible and of codimension 1. Our aim is to study the fundamental group of its complement, which we write as

$$\Phi = \pi_1\big(\mathbb{P}^N - \Delta\big).$$

When we need to make precise statements, we sometimes write $\Phi_{d,n} = \pi_1(U_{d,n}, o)$, where d and n are as above, $U_{d,n} = \mathbb{P}^N - \Delta$, and o is a base point.

The groups Φ are almost always nontrivial and, in fact, are almost always *large*; that is, there is a homomorphism of Φ to a noncompact semisimple real algebraic group which has Zariski-dense image. Large groups are infinite and, moreover, always contain a free group of rank 2. This follows from the Tits alternative [35], which states that in characteristic zero a linear group either has a solvable subgroup of finite index or contains a free group of rank 2.

To show that $\Phi = \Phi_{d,n}$ is large, we consider the image $\Gamma = \Gamma_{d,n}$ of the monodromy representation

$$\rho : \Phi \longrightarrow G. \tag{1.2}$$

Here and throughout this paper, $G = G_{d,n}$ denotes the group of automorphisms of the primitive cohomology $H^n(X_o, \mathbb{R})_o$ that preserve the cup product. When n is odd, the primitive cohomology is the same as the cohomology; when n is even, it is the

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